

Higgs-mediated FCNCs: Natural Flavour Conservation vs. Minimal Flavour Violation

Andrzej J. Buras^{a,b}, Maria Valentina Carlucci^a,
Stefania Gori^{a,c}, Gino Isidori^{b,d}

^a*Physik-Department, Technische Universität München, James-Frank-Straße,
D-85748 Garching, Germany*

^b*TUM Institute for Advanced Study, Technische Universität München, Arcisstraße 21,
D-80333 München, Germany*

^c*Max-Planck-Institut für Physik (Werner-Heisenberg-Institut),
D-80805 München, Germany*

^d*INFN, Laboratori Nazionali di Frascati, Via E. Fermi 40, I-00044 Frascati, Italy*

Abstract

We compare the effectiveness of two hypotheses, Natural Flavour Conservation (NFC) and Minimal Flavour Violation (MFV), in suppressing the strength of flavour-changing neutral-currents (FCNCs) in models with more than one-Higgs doublet. We show that the MFV hypothesis, in its general formulation, is more stable in suppressing FCNCs than the hypothesis of NFC alone when quantum corrections are taken into account. The phenomenological implications of the two scenarios are discussed analysing meson-antimeson mixing observables and the rare decays $B_{s,d} \rightarrow \mu^+ \mu^-$. We demonstrate that, introducing *flavour-blind* CP phases, two-Higgs doublet models respecting the MFV hypothesis can accommodate a large CP-violating phase in B_s mixing, as hinted by CDF and D0 data and, without extra free parameters, soften significantly in a correlated manner the observed anomaly in the relation between ε_K and $S_{\psi K_S}$.

1 Introduction

The standard assignment of the $SU(2)_L \times U(1)_Y$ quark charges, identified long ago by Glashow, Iliopoulos, and Maiani (GIM) [1], forbids tree-level flavour-changing couplings of the quarks to the Standard Model (SM) neutral gauge bosons. In the case of only one-Higgs doublet, namely within the SM, this structure is effective also in eliminating a possible dimension-four flavour-changing neutral-current (FCNC) coupling of the quarks to the Higgs field. While the $SU(2)_L \times U(1)_Y$ assignment of quarks and leptons can be considered as being well established, much less is known about the Higgs sector of the theory. In the presence of more than one-Higgs field the appearance of tree-level FCNC is not automatically forbidden by the standard assignment of the $SU(2)_L \times U(1)_Y$ fermion charges: additional conditions have to be imposed on the model in order to guarantee a sufficient suppression of FCNC processes [2, 3]. The absence

of renormalizable couplings contributing at the tree-level to FCNC processes, in multi-Higgs models, goes under the name of Natural Flavour Conservation (NFC) hypothesis.

The idea of NFC has been with us for more than 30 years. During the last decade another concept for the suppression of FCNC processes has become very popular: the hypothesis of Minimal Flavour Violation (MFV) [4, 5], whose origin, in specific new-physics (NP) models, can be traced back to [6, 7]. The question then arises how NFC (and GIM) are related to MFV, and vice versa. Motivated by a series of recent studies about the strengths of FCNCs in multi-Higgs doublet models [8–12], in this paper we present a detailed analysis of the relation between the NFC and MFV hypotheses. As we will show, while the two hypotheses are somehow equivalent at the tree-level, important differences arise when quantum corrections are included. Beyond the tree level, or beyond the implementation of these two hypotheses in their simplest version, some FCNCs are naturally generated in both cases. In this more general framework, the MFV hypothesis in its general formulation [5] turns out to be more stable in suppressing FCNCs than the hypothesis of NFC alone.

This analysis will also give us the opportunity to compare the various formulations of MFV present in the literature and to clarify which of the multi-Higgs models proposed in [8–10] are consistent with the MFV principle, and thus are naturally protected against too large FCNCs.

The phenomenological tests of these different concepts which can be obtained on the basis of meson-antimeson mixing observables, such as the CP-violating (CPV) observable ε_K , the mass differences $\Delta M_{d,s}$, and the CP asymmetries $S_{\psi K_S}$ and $S_{\psi\phi}$ are also analysed. Beside being perfectly consistent with present data even for light Higgs boson masses, two-Higgs doublet models respecting the MFV hypothesis could even accommodate a large CP-violating phase in B_s mixing, as hinted by CDF [13] and D0 [14, 15]. However, as pointed out first in [16], this can happen only introducing flavour-blind phases, i.e. decoupling the breaking of the flavour group from the breaking of the CP symmetry [16–18]. We demonstrate that, introducing flavour-blind CPV phases, such models¹ are not only capable of accommodating a large CPV phase in B_s mixing: also the observed anomaly in the relation between ε_K and $S_{\psi K_S}$ [19, 20] is substantially softened in a strictly correlated manner. We finally stress the key role of $B_{s,d} \rightarrow \ell^+ \ell^-$ decays in providing a future clean experimental tests of the MFV hypothesis in the Higgs sector, independently of possible flavour-blind phases.

The paper is organized as follows. In Section 2 we define the two hypotheses of NFC and MFV starting from the general quark Yukawa coupling with two-Higgs doublets. In Section 3 we analyse the problems of implementing NFC beyond the tree-level. The stability of MFV beyond the lowest order, and the comparison with the previous literature, is presented in Section 4 and 5, respectively. The phenomenological tests of $2\text{HDM}_{\overline{\text{MFV}}}$ by means of ε_K , $\Delta M_{s,d}$, $S_{\psi K_S}$, $S_{\psi\phi}$, and $B_{s,d} \rightarrow \ell^+ \ell^-$ decays are discussed in Section 6. We close our paper with a list of main lessons obtained through our analysis. Some technical details on the Higgs potential and our notations can be found in an Appendix.

¹The concrete two-Higgs doublet model belonging to this class will be called $2\text{HDM}_{\overline{\text{MFV}}}$ with the "bar" signalling the presence of flavour-blind CPV phases.

2 NFC and MFV hypotheses: definition and implementation to lowest order

Let's consider a model with two-Higgs doublets, H_1 and H_2 , with hypercharges $Y = 1/2$ and $Y = -1/2$, respectively. The most general renormalizable and gauge-invariant interaction of these fields with the SM quarks is

$$-\mathcal{L}_Y^{\text{gen}} = \bar{Q}_L X_{d1} D_R H_1 + \bar{Q}_L X_{u1} U_R H_1^c + \bar{Q}_L X_{d2} D_R H_2^c + \bar{Q}_L X_{u2} U_R H_2 + \text{h.c.} , \quad (1)$$

where $H_{1(2)}^c = -i\tau_2 H_{1(2)}^*$ and the X_i are 3×3 matrices with a generic flavour structure. The quark mass matrices are linear combinations of the matrices X_i , weighted by the corresponding Higgs vacuum expectation values (vevs):

$$M_d = \frac{1}{\sqrt{2}} (v_1 X_{d1} + v_2 X_{d2}) , \quad M_u = \frac{1}{\sqrt{2}} (v_1 X_{u1} + v_2 X_{u2}) . \quad (2)$$

Here $\langle H_{1(2)}^\dagger H_{1(2)} \rangle = v_{1(2)}^2/2$, with $v^2 = v_1^2 + v_2^2 \approx (246 \text{ GeV})^2$ and, by means of global phase transformations of $H_{1,2}$, we have eliminated possible CPV phases in the Higgs vevs (i.e. we have shifted CPV phases in the Higgs interaction terms). For generic X_i we cannot diagonalize simultaneously these two mass matrices and the couplings to the three physical neutral Higgs fields. Consequently we are left with dangerous FCNC couplings to some of them.

The X_i break in different ways the large quark-flavour symmetry of the gauge sector of the SM. They also break possible continuous or discrete symmetries associated to the Higgs sector. A convenient classification of various two-Higgs doublet models, and of the possible protection of FCNCs is obtained by identifying how these symmetries are broken. For simplicity, we focus the following discussion only on the quark sector of a two-Higgs doublet model (2HDM), but the analysis can easily be generalized to include the lepton sector and more than two-Higgs doublets.

The largest group of unitary quark field transformations that commutes with the SM gauge Lagrangian can be decomposed as [5, 6],

$$\mathcal{G}_q = \text{SU}(3)_q^3 \otimes \text{U}(1)_B \otimes \text{U}(1)_Y \otimes \text{U}(1)_{\text{PQ}} , \quad (3)$$

where

$$\text{SU}(3)_q^3 = \text{SU}(3)_{Q_L} \otimes \text{SU}(3)_{U_R} \otimes \text{SU}(3)_{D_R} \quad (4)$$

and the three $\text{U}(1)$ symmetries are the baryon number, the hypercharge, and the Peccei-Quinn symmetry [21], respectively. As far as $\text{U}(1)_{\text{PQ}}$ is concerned, we define it as the symmetry under which D_R and H_1 have opposite charge, while all the other fields are neutral. Since we assume that hypercharge is not explicitly broken, and that baryon number is conserved, the two ingredients in the classification of the structure of the Yukawa interaction are:

- the breaking of the flavour-blind $\text{U}(1)_{\text{PQ}}$ symmetry and of other discrete flavour-blind symmetries involving both right-handed quarks and Higgs fields;
- the breaking of the $\text{SU}(3)_q^3$ flavour symmetry.

According to which of these two breaking mechanism is *protected*, we can identify the two frameworks we are interested in:

- The *Natural Flavour Conservation* hypothesis, formulated in [2], is the assumption that only one-Higgs field can couple to a given quark species. This structure can be implemented by appropriate *flavour-blind* symmetries. In particular, the so-called type-II model, namely the condition

$$X_{u1} = X_{d2} = 0 \quad [\text{NFC, Type - II}] , \quad (5)$$

is obtained requiring the invariance of $\mathcal{L}_Y^{\text{gen}}$ under $U(1)_{\text{PQ}}$. The same result can be obtained using a discrete subgroup of $U(1)_{\text{PQ}}$: the Z_2 symmetry under which $H_1 \rightarrow -H_1$, $D_R \rightarrow -D_R$ and all other fields are unchanged. Another realization of the NFC hypothesis is the so-called type-I model, namely the condition

$$X_{u2} = X_{d2} = 0 \quad [\text{NFC, Type - I}] , \quad (6)$$

that can be obtained imposing the Z_2 symmetry under which only $H_2 \rightarrow -H_2$ and all other fields are unchanged.

- The *Minimal Flavour Violation* hypothesis, as formulated in [5], is the assumption that the $SU(3)_q^3$ flavour symmetry is broken only by two independent terms, Y_d and Y_u , transforming as

$$Y_u \sim (3, \bar{3}, 1)_{SU(3)_q^3} , \quad Y_d \sim (3, 1, \bar{3})_{SU(3)_q^3} . \quad (7)$$

Expanding to the lowest non-trivial order in these breaking terms leads to the following structure for the X_i couplings:

$$\begin{aligned} X_{d1} &= c_{d1} Y_d & X_{d2} &= c_{d2} Y_d \\ X_{u1} &= c_{u1} Y_u & X_{u2} &= c_{u2} Y_u \end{aligned} \quad [\text{MFV, } \mathcal{O}(Y^1)] , \quad (8)$$

where the $Y_{u,d}$ are 3×3 matrices and the c_i are arbitrary (flavour-blind) coefficients. If the breaking of the $SU(3)_q^3$ flavour group and the breaking of CP are decoupled, i.e. if we allow the introduction of flavour-blind phases in the MFV framework [16–18], the c_i coefficients in Eq. (8) can be complex.

The structure in Eq. (8), with complex c_i , has recently been postulated by Pich and Tuzon in Ref. [9]. These authors introduced this structure as an alternative to discrete symmetries in the Higgs sector to avoid FCNCs in a general two-Higgs doublet model. Here we have shown that this ansatz can be straightforwardly derived from the MFV hypothesis about the breaking of the $SU(3)_q^3$ flavour group [5], generalized to include flavour-blind phases [16–18], in the limit where the expansion in the $SU(3)_q^3$ breaking terms is truncated to the first order. As we will discuss in the following, the MFV hypothesis is a key ingredient to make this ansatz sufficiently stable beyond the tree-level when two-Higgs doublet model is considered to be only a low-energy effective theory (as expected by naturalness arguments).

To explicitly check that the conditions in Eq. (8) lead to the absence of FCNCs at tree-level, it is convenient to change the basis for the Higgs fields, moving to the basis where only one-Higgs doublet has a non-vanishing vev (see Appendix). This is achieved by the rotation

$$\begin{pmatrix} \Phi_v \\ \Phi_H \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} H_1 \\ H_2^c \end{pmatrix} , \quad c_\beta = \frac{v_1}{v} , \quad s_\beta = \frac{v_2}{v} , \quad t_\beta = \frac{s_\beta}{c_\beta} , \quad (9)$$

such that $\langle \Phi_v^\dagger \Phi_v \rangle = v^2/2$ and $\langle \Phi_H^\dagger \Phi_H \rangle = 0$. In this basis the Yukawa Lagrangian assumes the form

$$-\mathcal{L}_Y^{\text{gen}} = \bar{Q}_L \left[\frac{\sqrt{2}}{v} M_d \Phi_v + Z_d \Phi_H \right] D_R + \bar{Q}_L \left[\frac{\sqrt{2}}{v} M_u \Phi_v^c + Z_u \Phi_H^c \right] U_R + \text{h.c.} , \quad (10)$$

with the 3×3 matrices $Z_{d,u}$ given by

$$Z_d = c_\beta X_{d2} - s_\beta X_{d1} , \quad Z_u = c_\beta X_{u2} - s_\beta X_{u1} . \quad (11)$$

It is then straightforward to check that $Z_d \propto M_d$ and $Z_u \propto M_u$ for all cases in Eqs. (5), (6), and (8). This implies that quark mass terms and couplings to the neutral Higgs fields can be diagonalized simultaneously, resulting in the absence of tree-level FCNCs. Note also that the two NFC structures in Eq. (5) and (6) correspond to specific limits for the c_i coefficients of the linear MFV structure in Eq. (8), as recently pointed out in Ref. [9]. Less trivial is to understand how these Yukawa interactions get modified after the inclusion of quantum corrections. This is the subject of the next two sections.

3 The problems of NFC beyond the lowest order

As discussed in the previous section, the NFC hypothesis can be enforced by means of appropriate *flavour-blind* symmetries. As we will show in the following, these symmetries alone are not sufficient to protect the effective Yukawa interaction beyond the lowest order.

3.1 Breaking of $U(1)_{PQ}$ beyond the tree-level

We consider first the case where the type-II structure in Eq. (5) is enforced by means of the $U(1)_{PQ}$ symmetry. In this case the structure is not stable since this continuous symmetry must be explicitly broken in other sectors of the theory in order to avoid a massless pseudoscalar Higgs field. The $U(1)_{PQ}$ breaking will then induce non-vanishing X_{u1} and X_{d2} beyond the tree-level. If the underlying theory contains additional sources of flavour symmetry breaking beside the quark Yukawa couplings (i.e. if the theory is not compatible with the MFV hypothesis), the loop-induced couplings X_{u1} and X_{d2} may lead to very large FCNCs. This is for instance what happens in the minimal supersymmetric extension of the SM (MSSM) with generic soft-breaking terms [22, 23].²

To quantify the amount of fine-tuning in this scenario in the presence of $U(1)_{PQ}$ breaking but not imposing MFV, we consider in detail the case of the down-type Yukawa coupling. After the breaking of the NFC relation, X_{d1} and X_{d2} can be decomposed as

$$X_{d1} = Y_d , \quad X_{d2} = \epsilon_d \Delta_d , \quad (12)$$

where Δ_d is a generic 3×3 flavour-breaking matrix, with $\mathcal{O}(1)$ entries, and ϵ_d is a real parameter controlling the size of the $U(1)_{PQ}$ breaking. Since the breaking of $U(1)_{PQ}$ is generated only beyond the tree-level, we can assume $\epsilon_d \ll 1$. In the basis where Y_d is diagonal the down-type mass matrix in (2) assumes the form

$$(M_d)_{ij} = \frac{v_1}{\sqrt{2}} \left[(Y_d^{\text{eff}})_{ii} \delta_{ij} + \epsilon_d t_\beta (\tilde{\Delta}_d)_{ij} \right] , \quad (13)$$

² In the absence of an explicit breaking, the $U(1)_{PQ}$ symmetry would be spontaneously broken by the vev of H_1 , hence the theory would contain a Goldstone boson. This problem can be avoided, and $U(1)_{PQ}$ does not need to be explicitly broken, if the vev of H_1 is zero (see e.g. Ref. [24]). However, also in this case the smallness of FCNCs is not guaranteed beyond the tree level (in the absence of MFV) because of the argument presented in Sect. 3.3.

where $(Y_d^{\text{eff}})_{ii} = (Y_d)_{ii} + \epsilon_d t_\beta (\Delta_d)_{ii}$ and $\tilde{\Delta}_d$ is the off-diagonal part of Δ_d . We can then proceed with a perturbative diagonalization of M_d to first order in ϵ_d . This is obtained via the rotations

$$Q_L^i \rightarrow \left[\delta_{ij} + \epsilon_d t_\beta \frac{(\tilde{\Delta}_d)_{ij} (Y_d^{\text{eff}})_{jj} + (\tilde{\Delta}_d)_{ji}^* (Y_d^{\text{eff}})_{ii}}{(Y_d^{\text{eff}})_{jj}^2 - (Y_d^{\text{eff}})_{ii}^2} \right] Q_L^j, \quad (14)$$

$$D_R^i \rightarrow \left[\delta_{ij} + \epsilon_d t_\beta \frac{(\tilde{\Delta}_d)_{ji}^* (Y_d^{\text{eff}})_{jj} + (\tilde{\Delta}_d)_{ij} (Y_d^{\text{eff}})_{ii}}{(Y_d^{\text{eff}})_{jj}^2 - (Y_d^{\text{eff}})_{ii}^2} \right] D_R^j. \quad (15)$$

In the basis where M_d is diagonal the effective coupling Z_d defined in Eq. (11) assumes the form

$$(Z_d)_{ij} = (Z_d^{\text{diag}})_{ii} \delta_{ij} + \frac{\epsilon_d}{c_\beta} (\tilde{\Delta}_d)_{ij}, \quad (Z_d^{\text{diag}})_{ii} = -s_\beta (Y_d)_{ii} + c_\beta \epsilon_d (\Delta_d)_{ii}, \quad (16)$$

which implies the following FCNC coupling:

$$\mathcal{L}_\epsilon^{\text{FCNC}} = -\frac{\epsilon_d}{c_\beta} (\tilde{\Delta}_d)_{ij} \bar{d}_L^i d_R^j \frac{S_2 + iS_3}{\sqrt{2}} + \text{h.c.}, \quad (17)$$

where $S_{2,3}$ are the neutral components of the Higgs doublet with vanishing vev (see Appendix). For $(\tilde{\Delta}_d)_{ij} = \mathcal{O}(1)$ and $\epsilon_d = \mathcal{O}(10^{-2})$, as expected by a typical loop suppression, this effective coupling is well above the experimental bounds on FCNCs. In particular, it largely exceeds the bounds from CP-violation in $K^0 - \bar{K}^0$ mixing.

3.2 The ϵ_K bound on generic scalar FCNCs

To evaluate the impact of the FCNC coupling in Eq. (17), we consider the simplifying case where the mass mixing between Φ_H and Φ_v , and possible CP-violating terms in the Higgs potential can be neglected (the so-called decoupling limit, that is naturally realized for $t_\beta \gg 1$, see Appendix). In this limit the neutral components of Φ_H are the CP-even and CP-odd mass-eigenstates H^0 and A^0 , with degenerate mass M_H .

Integrating out the heavy Higgs fields at the tree-level leads to the following $\Delta S = 2$ effective Hamiltonian

$$\mathcal{H}_\epsilon^{|\Delta S|=2} = -\frac{\epsilon_d^2}{c_\beta^2 M_H^2} (\tilde{\Delta}_d)_{21} (\tilde{\Delta}_d)_{12}^* (\bar{s}_L d_R) (\bar{s}_R d_L) + \text{h.c.} \quad (18)$$

Taking into account the large QCD corrections in the evolution from $\mu \sim M_H$ down to a scale $\mu_K \sim 2$ GeV, this effective Hamiltonian implies a potentially sizable non-standard contribution to ϵ_K (see Section 6.1). Imposing the condition $|\epsilon_K^{\text{NP}}| < 0.2 |\epsilon_K^{\text{exp}}|$, to be in agreement with experiment, leads to the bound

$$|\epsilon_d| \times \left| \text{Im}[(\tilde{\Delta}_d)_{21}^* (\tilde{\Delta}_d)_{12}] \right|^{1/2} \lesssim 3 \times 10^{-7} \times \frac{c_\beta M_H}{100 \text{ GeV}}. \quad (19)$$

This result illustrates the large amount of fine-tuning needed on ϵ_d if the new flavour-breaking matrix Δ_d has entries of $\mathcal{O}(1)$: a loop suppression of $\mathcal{O}(10^{-2})$ on ϵ_d is not enough to avoid a huge contribution to ϵ_K , if $M_H \lesssim 1$ TeV. In other words, we *cannot avoid an efficient protection of the flavour structure*, if we want to avoid too large FCNCs. One could of course suppress FCNCs choosing a very large value for M_H , but this would introduce a fine-tuning problem in the Higgs sector.

The MFV hypothesis is not the only allowed possibility to reach a sufficiently small breaking of the $SU(3)_q^3$ flavour symmetry. For instance, in models with warped space-time geometry [25] or, equivalently, models with partial compositeness [26], or hierarchical fermion wave functions [27], we expect

$$|(\tilde{\Delta}_d)_{ij}^*(\tilde{\Delta}_d)_{ji}|_{\text{RS-GIM}} = \mathcal{O}(1) \times [(Y_d)_{ii}(Y_d)_{jj}] = \mathcal{O}(1) \times \frac{2m_{d_i}m_{d_j}}{c_\beta^2 v^2} , \quad (20)$$

where the quark masses have to be evaluated at the scale $\mu \sim M_H$. Using the above relation to set a bound on $\text{Im}[(\tilde{\Delta}_d)_{21}^*(\tilde{\Delta}_d)_{12}]$, with the unknown $\mathcal{O}(1)$ coefficients fixed to 1, leads to

$$|\epsilon_d|_{\text{RS-GIM}} \lesssim 4 \times 10^{-3} \times \frac{c_\beta^2 M_H}{100 \text{ GeV}} . \quad (21)$$

In this case a $\mathcal{O}(10^{-2})$ suppression on ϵ_d and a not too light M_H could be sufficient to avoid the ε_K bound, but only if $t_\beta = \mathcal{O}(1)$. If t_β is large, then also in this case a non-negligible amount of fine-tuning is needed. Moreover, once the ε_K bound is enforced, non-standard effects in $\Delta B = 2$ amplitudes are naturally suppressed, unless some amount of fine-tuning on the $\mathcal{O}(1)$ coefficients (the five-dimensional Yukawa couplings) is introduced [28].

As we will show in Section 4, the picture is quite different in the MFV case. Within the MFV framework large values of t_β cannot be excluded, and the most interesting phenomenology is expected in the $B_{s,d}$ -meson systems.

3.3 Discrete symmetries and higher-dimensional Yukawa-type interactions

To derive the effective FCNC coupling in Eq. (17) we assumed that the type-II NFC structure of the dimension-four Yukawa couplings is violated at the quantum level as a consequence of the breaking of the $U(1)_{PQ}$ symmetry. If the symmetry used to enforce the NFC structure is a discrete one, this is not necessarily true: we can conceive a NFC scenario where $U(1)_{PQ}$ is explicitly broken, while the Z_2 symmetry $H_1 \rightarrow -H_1$, $D_R \rightarrow -D_R$ is exact. In this case X_{d2} and X_{u1} are strictly zero. However, also this condition is not sufficient to protect FCNCs if the theory has additional degrees of freedom at the TeV scale, as expected by a natural stabilization of the mechanism of electroweak symmetry breaking.

Integrating out the heavy fields at the TeV scale in a Z_2 invariant framework generates higher-dimensional operators of the type

$$\begin{aligned} \Delta\mathcal{L}_Y = & \frac{c_1}{\Lambda^2} \bar{Q}_L X_{u1}^{(6)} U_R H_2 |H_1|^2 + \frac{c_2}{\Lambda^2} \bar{Q}_L X_{u2}^{(6)} U_R H_2 |H_2|^2 \\ & + \frac{c_3}{\Lambda^2} \bar{Q}_L X_{d1}^{(6)} D_R H_1 |H_1|^2 + \frac{c_4}{\Lambda^2} \bar{Q}_L X_{d2}^{(6)} D_R H_1 |H_2|^2 , \end{aligned} \quad (22)$$

with $c_i = \mathcal{O}(1)$ and $\Lambda = \mathcal{O}(1 \text{ TeV})$. These operators are Z_2 invariant. However, after the Higgs fields get a vev, they break the proportionality relation between quark mass terms and effective interaction with the neutral scalars, even in the case of a single Higgs doublet [29–31]. As a result, after the mass diagonalization, $\Delta\mathcal{L}_Y$ leads to effective FCNC couplings of the type in Eq. (17), where also the physical component of the Φ_v doublet appears.

In this context the role of the Peccei-Quinn symmetry breaking term ϵ_d is replaced by a parameter of order v^2/Λ^2 . From the bound in Eq. (19), it is clear that for $\Lambda = \mathcal{O}(1 \text{ TeV})$ this suppression is not sufficient to be in agreement with data, unless the flavour structure of the $X_i^{(6)}$ is sufficiently protected.

4 Stability of MFV beyond the lowest order

4.1 General considerations

The general structure implied by the MFV hypothesis for the renormalizable Yukawa couplings defined in (1) is

$$X_{d1} = P_{d1}(Y_u Y_u^\dagger, Y_d Y_d^\dagger) \times Y_d , \quad (23)$$

$$X_{d2} = P_{d2}(Y_u Y_u^\dagger, Y_d Y_d^\dagger) \times Y_d , \quad (24)$$

$$X_{u1} = P_{u1}(Y_u Y_u^\dagger, Y_d Y_d^\dagger) \times Y_u , \quad (25)$$

$$X_{u2} = P_{21}(Y_u Y_u^\dagger, Y_d Y_d^\dagger) \times Y_u , \quad (26)$$

where $P_i(Y_u Y_u^\dagger, Y_d Y_d^\dagger)$ are generic polynomials of the two basic left-handed spurions

$$Y_u Y_u^\dagger, Y_d Y_d^\dagger \sim (8, 1, 1)_{\text{SU}(3)_q^3} \oplus (1, 1, 1)_{\text{SU}(3)_q^3} . \quad (27)$$

Since we are free to re-define the two basic spurions Y_u and Y_d , without loss of generality we can define them to be the flavour structures appearing in X_{d1} and X_{u2} . Then expanding the remaining non-trivial polynomials in powers of $Y_u^\dagger Y_u$ and $Y_d^\dagger Y_d$ we get

$$\begin{aligned} X_{d1} &= Y_d , \\ X_{d2} &= \epsilon_0 Y_d + \epsilon_1 Y_d Y_d^\dagger Y_d + \epsilon_2 Y_u Y_u^\dagger Y_d + \dots , \\ X_{u1} &= \epsilon'_0 Y_u + \epsilon'_1 Y_u Y_u^\dagger Y_u + \epsilon'_2 Y_d Y_d^\dagger Y_u + \dots , \\ X_{u2} &= Y_u . \end{aligned} \quad (28)$$

This structure, which has been considered first in full generality in Ref. [5], is renormalization group (RG) invariant. It is the most general form compatible with the breaking of the flavour group $SU(3)_q^3$ by the two spurions in Eq. (7). Quantum corrections can change the values of the ϵ_i at different energy scales, but they cannot modify this functional form.³ We stress that this functional form is respected only if the full theory, including possible high-energy degrees of freedom, respects the MFV principle. On the contrary, if we start from the linear structure in Eq. (8) but we do not assume the MFV principle, we may end up with the problems discussed in the previous section when going beyond the tree-level.

In principle, the series in Eq. (28) contain an infinite number of terms. However, barring fine-tuned scenarios where Y_d and Y_u have a structure substantially different than what determined in the one-Higgs case, we can still perform the usual MFV expansion in powers of suppressed off-diagonal CKM elements. More explicitly, with an appropriate rotation of the quark fields we can always choose a basis such that

$$\begin{aligned} Y_d &\xrightarrow{\text{d-basis}} \text{diag}(\hat{y}_d, \hat{y}_s, \hat{y}_b) \equiv \hat{\lambda}_d , \\ Y_u &\xrightarrow{\text{d-basis}} \hat{V}^\dagger \times \text{diag}(\hat{y}_u, \hat{y}_c, \hat{y}_t) \equiv \hat{V}^\dagger \hat{\lambda}_u , \end{aligned} \quad (29)$$

³ The Yukawa expansion in Eq. (28) is very similar to the expansion of the soft-breaking terms in the MSSM, for which explicit studies of RG equations in the MFV framework have been discussed in Ref. [32, 33].

where the hat over V and the Yukawa eigenvalues distinguish them from the “standard” values obtained in the $\epsilon_i^{(i)} = 0$ limit:

$$\hat{\lambda}_d \xrightarrow{\epsilon_i^{(i)}=0} \lambda_d = \text{diag}(y_d, y_s, y_b) , \quad y_{d_i} = \sqrt{2}m_{d_i}/v_1 , \quad (30)$$

$$\hat{\lambda}_u \xrightarrow{\epsilon_i^{(i)}=0} \lambda_u = \text{diag}(y_u, y_c, y_t) , \quad y_{u_i} = \sqrt{2}m_{u_i}/v_2 , \quad (31)$$

$$\hat{V} \xrightarrow{\epsilon_i^{(i)}=0} V [= \text{CKM matrix}] . \quad (32)$$

While we cannot fully determine the \hat{V} and $\hat{\lambda}_{d,u}$ without knowing the values of the $\epsilon_i^{(i)}$, the smallness of the off-diagonal elements of \hat{V} and of the Yukawa eigenvalues of the first two generations is parametrically stable, even for values of $\epsilon_i^{(i)}t_\beta = \mathcal{O}(1)$. As a result, the only large entries in the series (28) are those involving flavour-diagonal entries of the third generation and we are left with only two relevant basic spurions in the basis (29). Adopting the notation of Ref. [5] we define them as

$$\Delta = \frac{1}{\hat{y}_b^2} Y_d Y_d^\dagger \approx \text{diag}(0, 0, 1) , \quad (\hat{\lambda}_{\text{FC}})_{ij} = \begin{cases} (Y_u Y_u^\dagger)_{ij} \approx \hat{y}_t^2 \hat{V}_{3i}^* \hat{V}_{3j} & i \neq j , \\ 0 & i = j . \end{cases} \quad (33)$$

Expanding to first non-trivial order in these two spurions we get [5]

$$X_{d2} = \left(\epsilon_0 + \epsilon_1 \Delta + \epsilon_2 \hat{\lambda}_{\text{FC}} + \epsilon_3 \hat{\lambda}_{\text{FC}} \Delta + \epsilon_4 \Delta \hat{\lambda}_{\text{FC}} \right) \hat{\lambda}_d , \quad (34)$$

$$X_{u1} = \left(\epsilon'_0 + \epsilon'_1 \Delta + \epsilon'_2 \hat{\lambda}_{\text{FC}} + \epsilon'_3 \hat{\lambda}_{\text{FC}} \Delta + \epsilon'_4 \Delta \hat{\lambda}_{\text{FC}} \right) \hat{V}^\dagger \hat{\lambda}_u . \quad (35)$$

We stress that this form is not a simple linear expansion in the Yukawa couplings, rather an expansion in the small terms associated to off-diagonal CKM matrix elements and light quark masses. The resummation to all orders of high-powers of \hat{y}_t^2 or \hat{y}_b^2 , whose importance has been stressed in [16, 34], is implicitly taken into account by a redefinition of the $\epsilon_i^{(i)}$ parameters. Throughout this paper we also assume the $\epsilon_i^{(i)}$ are small ($\epsilon_i^{(i)} < 1$) as resulting from an approximate $U(1)_{\text{PQ}}$ symmetry. Beside its phenomenological interest, this assumption allows us to unambiguously define t_β starting from the $\epsilon_i^{(i)} \rightarrow 0$ limit (see Appendix).

The diagonalization of the quark mass matrices keeping the $\epsilon_i t_\beta$ terms to all orders (assuming real ϵ_i and neglecting ϵ'_i/t_β) has been presented in Ref. [5] (see also [35–38]) and will not be repeated here. The main results can be summarized as follows:

- The relation between λ_d and $\hat{\lambda}_d$ is

$$\lambda_d = [1 + (\epsilon_0 + \epsilon_1 \Delta)t_\beta] \hat{\lambda}_d , \quad (36)$$

while the up-type mass matrix remains unaffected ($\hat{\lambda}_u = \lambda_u$) in the limit where we neglect $\mathcal{O}(\epsilon'_i/t_\beta)$ terms. The physical CKM matrix coincides with \hat{V} but for the V_{i3} and V_{3i} entries ($i \neq 3$) for which

$$\frac{\hat{V}_{i3}}{V_{i3}} = \frac{\hat{V}_{3i}}{V_{3i}} = 1 + r_V , \quad r_V \equiv \frac{(\epsilon_2 + \epsilon_3)t_\beta}{1 + (\epsilon_0 + \epsilon_1 - \epsilon_2 - \epsilon_3)t_\beta} . \quad (37)$$

- The diagonalization of the mass terms does not eliminate scalar FCNC interactions. In the case of down-type quarks, the effective FCNC coupling surviving after the diagonalization can be written as

$$\mathcal{L}_{\text{MFV}}^{\text{FCNC}} = -\frac{1}{s_\beta} \bar{d}_L^i \left[\left(a_0 V^\dagger \lambda_u^2 V + a_1 V^\dagger \lambda_u^2 V \Delta + a_2 \Delta V^\dagger \lambda_u^2 V \right) \lambda_d \right]_{ij} d_R^j \frac{S_2 + iS_3}{\sqrt{2}} + \text{h.c.}, \quad (38)$$

where $S_{2,3}$ are the neutral components of the Higgs doublet with no vev (Φ_H , see also Appendix), and

$$\begin{aligned} a_0 &= \frac{\epsilon_2 t_\beta (1 + r_V)^2}{y_t^2 [1 + \epsilon_0 t_\beta]^2}, & a_1 + a_0 &= \frac{r_V}{y_t^2 [1 + (\epsilon_0 + \epsilon_1) t_\beta]}, \\ a_2 - a_1 &= \frac{(\epsilon_4 - \epsilon_3) t_\beta}{y_t^2 [1 + \epsilon_0 t_\beta] [1 + (\epsilon_0 + \epsilon_1 - \epsilon_2 - \epsilon_3) t_\beta]}. \end{aligned} \quad (39)$$

In principle, a FCNC coupling with the Φ_H doublet survives also in the up sector. However, this effect is less interesting since in this case the a'_i coefficients (defined in analogy with the a_i) turn out to be $\mathcal{O}(\epsilon'_i)$ and not of $\mathcal{O}(\epsilon_i t_\beta)$ as in (39).

As can be noted, $\mathcal{L}_{\text{MFV}}^{\text{FCNC}}$ exhibits the typical MFV structure of FCNCs, where all the non-vanishing effects are driven by the large top-quark Yukawa coupling. This structure implies a strong suppression of FCNCs because of the smallness of the CKM elements $|V_{ts}|$ and $|V_{td}|$. As a result, the a_i can be of $\mathcal{O}(1)$ even for $t_\beta \gg 1$ and $M_H < 1$ TeV (detailed phenomenological bounds are presented in Section 6). However, the presence of the Δ spurion, which reflects the possibility of large bottom Yukawa coupling, implies a possible $\mathcal{O}(1)$ breaking of the correlation between FCNCs in the K and in the $B_{s,d}$ systems that holds in the SM and in MFV for $t_\beta = \mathcal{O}(1)$.

4.2 Introducing flavour-blind phases

So far, following Ref. [5], we have assumed that the ϵ_i are real. This assumption is justified by the strong bounds on flavour-conserving CPV phases implied by the electric dipole moments⁴. However, this phenomenological problem could have a dynamical explanation and it is worth to investigate also the case of complex ϵ_i , namely the case where we allow generic CP-violating flavour-blind phases in the Higgs sector. Namely, we consider the generic framework where the Yukawa matrices are the only sources of breaking of the $SU(3)_q^3$ flavour group, but they are not the only allowed sources of CP-violation (a general discussion about this framework can be found in Ref. [16–18]).

As far as Higgs-mediated FCNCs are concerned, this amounts only to consider the a_i in Eq. (38) as complex parameters. Integrating out the neutral Higgs fields leads to tree-level contributions to scalar FCNC operators. Keeping complex a_i , and working in the decoupling limit for the heavy Higgs doublet (see Appendix), the leading $\Delta F = 1$ and $\Delta F = 2$ Hamiltonians

⁴As the recent analysis [40] shows, in the 2HDM $_{\text{MFV}}$ the electric dipole moments, although very much enhanced over the SM values, are still compatible with the experimental bounds.

thus generated are

$$\mathcal{H}_{\text{MFV}}^{|\Delta B|=1} = -\frac{a_0^* + a_1^*}{M_H^2} y_\ell y_b y_t^2 V_{tb}^* V_{tq} (\bar{b}_R q_L)(\bar{\ell}_L \ell_R) + \text{h.c.} \quad (q = d, s), \quad (40)$$

$$\mathcal{H}_{\text{MFV}}^{|\Delta S|=1} = -\frac{a_0^*}{M_H^2} y_\ell y_s y_t^2 V_{ts}^* V_{td} (\bar{s}_R d_L)(\bar{\ell}_L \ell_R) + \text{h.c.}, \quad (41)$$

$$\mathcal{H}_{\text{MFV}}^{|\Delta B|=2} = -\frac{(a_0^* + a_1^*)(a_0 + a_2)}{M_H^2} y_b y_q [y_t^2 V_{tb}^* V_{tq}]^2 (\bar{b}_R q_L)(\bar{b}_L q_R) + \text{h.c.} \quad (q = d, s), \quad (42)$$

$$\mathcal{H}_{\text{MFV}}^{|\Delta S|=2} = -\frac{|a_0|^2}{M_H^2} y_s y_d [y_t^2 V_{ts}^* V_{td}]^2 (\bar{s}_R d_L)(\bar{s}_L d_R) + \text{h.c.} \quad (43)$$

A detailed analysis of the phenomenological impact of these effective Hamiltonians is postponed to Section 6. We anticipate here two key properties that can be directly deduced by looking at their flavour- and CP-violating structure:

- I. The impact in $K^0-\bar{K}^0$, $B_d^0-\bar{B}_d^0$ and $B_s^0-\bar{B}_s^0$ mixing amplitudes scales, relative to the SM, with $m_s m_d$, $m_b m_d$ and $m_b m_s$, respectively. This fact opens the possibility of sizable non-standard contributions to the B_s system without serious constraints from $K^0-\bar{K}^0$ and $B_d^0-\bar{B}_d^0$ mixing.
- II. While the possible flavour-blind phases do not contribute to the $\Delta S = 2$ effective Hamiltonian, they could have an impact in the $\Delta B = 2$ case, offering the possibility to solve the recent experimental anomalies related to the B_s mixing phase. However, this happens only if the $\epsilon_{3,4}$ terms in the expansion (34) are at least as large as the other ϵ_i . This implies non-trivial underlying dynamical models, where effective operators with high powers of Yukawa insertions are not strongly suppressed (contrary to what happens, for instance, in the MSSM).

We emphasize that the CKM and quark-mass pattern of $\Delta F = 2$ amplitudes outlined above (point I.) is characteristic of the MFV pattern in its general formulation [5]. In particular, it differs from the so-called constrained MFV framework [4], where scalar amplitudes are negligible and the three down-type $\Delta F = 2$ amplitudes have a universal scaling relative to the SM (i.e. the scaling is dictated only by the CKM factors). It differs also from the RS-GIM (or hierarchical wave-function) framework [25–27], where we expect similar quark-mass suppression terms, but we do not expect also the CKM factors: as shown in Section 6.2.2, the RS-GIM scaling implies large effects in ε_K , relative to the SM, with minor corrections in the $B_{s,d}$ systems.

Within the general MFV framework this pattern is not necessarily a signal of Higgs-mediated effects, but it is a clear signal of non-Hermitian scalar operators of the type

$$(\bar{D}_R Y_d^\dagger Y_d Y_u^\dagger Y_u Q_L)(\bar{Q}_L Y_u Y_u^\dagger Y_d D_R), \quad (\bar{D}_R Y_d^\dagger Y_u Y_u^\dagger Q_L)(\bar{Q}_L Y_d Y_d^\dagger Y_u Y_u^\dagger D_R), \quad (44)$$

which should appear with a sizable coefficient and with large (and non identical) CP-violating phases.

5 Comparison with models in the literature

In this section we briefly comment about the flavour structure of the models recently proposed in [8–10], discussing which of them are compatible with the general MFV structure, as described in Section 2 and 4. To this purpose, it is first useful to clarify differences and similarities with

respect to Ref. [5] of other implementations of the MFV concept present in the literature and, more generally, of other constructions where the strength of FCNCs is related to the structure of the CKM matrix:

- The so-called *Constrained MFV*, originally proposed in [4], is based on the following two assumptions: 1) no new effective FCNC dimension-six operators beyond those already present in the SM; 2) the flavour and CPV structure of these operators is dictated by the CKM factors $V_{3i}^* V_{3j}$ (for $d_i \rightarrow d_j$ FCNC transitions), i.e. it is aligned in flavour space with the SM short-distance contribution. This proposal gives rise to a very predictive framework that coincides, in practice, with Ref. [5] in the limiting case of a single light Higgs field. However, by construction, the CMFV hypothesis cannot be applied to multi-Higgs models with large $\tan \beta$ where new operators, the scalar (left-right) operators, can be important.
- The so-called *General MFV*, proposed in [16], is based on the same symmetry and symmetry-breaking pattern proposed in [5], but for the decoupling of CP and flavour symmetries. In Ref. [16] particular emphasis is put on the fact that the expansion in powers of the top and bottom Yukawa couplings should not be truncated. As already discussed in Section 4, the non-linear structure associated to the third generation does not give rise to observable differences with respect to the formulation of [5], which is based only on the expansion in off-diagonal CKM elements $|V_{3i}|$ and light quark mass ratios m_{q_i}/m_{q_3} ($i \neq 3$). The only important difference between [16] and [5] is related to the possible introduction of flavour-blind phases, namely to the possible decoupling between the breaking of CP and flavour symmetries.
- The so-called *BGL models*, proposed in [39], is a class of two-Higgs doublet models where the strength of FCNCs in the up- or down-type sector is unambiguously related to the off-diagonal elements of the CKM matrix. While all the six BGL models are interesting, only one of them is compatible with the MFV principle. As can be deduced by looking at Eq. (33), only the BGL model where $d_i \rightarrow d_j$ FCNC transitions are proportional to $V_{3i}^* V_{3j}$ is an explicit example of MFV.

We are now ready to comment about the explicit models with more than one-Higgs doublet considered in Ref. [8–10]:

- As already pointed out in Section 2, the model of Ref. [9], denoted “Yukawa alignment model”, is a limiting case of the general MFV construction, where the higher-order powers in Y_u and Y_d are not included. As already pointed out, the higher-order terms are naturally generated by quantum corrections, but they don’t spoil the nice virtues of the MFV construction, once we assume that the MFV hypothesis is respected by the high-energy degrees of freedom of the theory. Being compatible with the MFV hypothesis, some phenomenological aspects of the model of Ref. [9] can be deduced as a limiting case from more general MFV analyses (see e.g. [5, 35–38]). However it must be stressed that, except for Ref. [9], previous works have not included the possibility of flavour-blind CP-violating phases, and have been focused mainly on neutral-Higgs effects.
- The first model considered in Ref. [10], namely a 2HDM with a non-trivial $U(1)$ symmetry for the third generation is an interesting explicit example of MFV. More precisely, this framework coincides with the MFV construction in the limit $m_{c,u}^2/m_t^2 \rightarrow 0$, which is an excellent approximation. On the other hand, the model with three-Higgs doublet

considered at the end of Ref. [10] is not compatible with the MFV principle and, not surprisingly, it leads to potentially too large effects in $K^0-\bar{K}^0$ mixing.

- As far as Ref. [8] is concerned, the model considered in the first paper is, in principle, compatible with MFV. However, the related phenomenological analysis is not reliable since the authors have assumed a huge $SU(2)_L$ breaking in the Higgs sector, which can be ruled out by electroweak precision tests. Indeed they analyse the phenomenological implications of the $\bar{d}_L^i d_R^j \bar{d}_L^i d_R^j$ operator that is forbidden in the exact $SU(2)_L$ limit (see Section 6.2). The model considered in the second paper in Ref. [8] is manifestly beyond MFV.

6 Phenomenological tests

6.1 $\Delta F = 2$ amplitudes: general discussion

In the general 2HDM the $\Delta F = 2$ transitions represented by the particle-antiparticle mass differences ΔM_K , $\Delta M_{s,d}$ and the CP-violating observables ε_K , $S_{\psi K_s}$ and $S_{\psi\phi}$ are governed by the SM box diagrams with up-quarks and W^\pm exchanges, the box diagrams with up-quarks and H^\pm exchanges and the tree-level neutral Higgs (h^0, H^0, A^0) exchanges.

When QCD renormalization group effects are taken into account, the following set of low-energy operators at scales $\mu_K \sim 2$ GeV in the case of $K^0 - \bar{K}^0$ mixing has to be taken into account:

$$\begin{aligned} Q_1^{VLL} &= (\bar{s}_L \gamma_\mu d_L)(\bar{s}_L \gamma^\mu d_L), & Q_1^{LR} &= (\bar{s}_L \gamma_\mu d_L)(\bar{s}_R \gamma^\mu d_R), \\ Q_1^{SLL} &= (\bar{s}_R d_L)(\bar{s}_R d_L), & Q_2^{LR} &= (\bar{s}_R d_L)(\bar{s}_L d_R), \\ Q_2^{SLL} &= (\bar{s}_R \sigma_{\mu\nu} d_L)(\bar{s}_R \sigma^{\mu\nu} d_L), \end{aligned} \quad (45)$$

where $\sigma_{\mu\nu} = \frac{1}{2} [\gamma_\mu, \gamma_\nu]$. In addition, the operators $Q_{1,2}^{SRR}$, analogous to $Q_{1,2}^{SLL}$ with the exchange $q_L \rightarrow q_R$, are also present. Once the Wilson coefficients of these operators $C_i(\mu_H)$ are calculated at a high energy scale, where heavy degrees of freedom are integrated out, the formulae in [41] allow automatically to calculate their values at scales $\mathcal{O}(\mu_K)$.

The resulting low energy effective Hamiltonian then reads

$$\mathcal{H}_{\text{eff}} = \sum_{i,a} C_i^a(\mu_K, K) Q_i^a, \quad (46)$$

where $i = 1, 2$, and $a = VLL, LR, SLL, SRR$. The off-diagonal element in $K^0 - \bar{K}^0$ mixing M_{12}^K is then given by

$$2M_K M_{12}^K = \langle \bar{K}^0 | \mathcal{H}_{\text{eff}} | K^0 \rangle^* \quad (47)$$

and the observables of interest can be directly evaluated. In particular

$$\Delta M_K = 2\text{Re}(M_{12}^K), \quad (48)$$

$$\varepsilon_K = e^{i\varphi_\varepsilon} \frac{\kappa_\varepsilon}{\sqrt{2} \Delta M_K} \text{Im}(M_{12}^K). \quad (49)$$

Here $\varphi_\varepsilon = (43.51 \pm 0.05)^\circ$ takes into account that $\varphi_\varepsilon \neq \pi/4$ and $\kappa_\varepsilon = 0.94 \pm 0.02$ [20, 42] includes an additional effect from long-distance contributions.

Finally for the $B_{s,d}$ systems, one has to evaluate the Wilson coefficients at scales $\mu_B \sim 4.2$ GeV. The computation of the CPV and CP-conserving (CPC) observables is then exactly

analogous to what we have just discussed for the K system. In particular, denoting with M_{12}^q the off-diagonal elements in the $B_q^0 - \bar{B}_q^0$ mixings, one has

$$\Delta M_q = 2 |M_{12}^q|, \quad (q = d, s). \quad (50)$$

Next we define

$$M_{12}^q = (M_{12}^q)_{\text{SM}} C_{B_q} e^{i2\varphi_{B_q}}, \quad (q = d, s), \quad (51)$$

where

$$(M_{12}^d)_{\text{SM}} = |(M_{12}^d)_{\text{SM}}| e^{2i\beta}, \quad \beta \approx 0.38, \quad (52)$$

$$(M_{12}^s)_{\text{SM}} = |(M_{12}^s)_{\text{SM}}| e^{2i\beta_s}, \quad \beta_s \simeq -0.01, \quad (53)$$

and the phases β and β_s are defined through

$$V_{td} = |V_{td}| e^{-i\beta} \quad \text{and} \quad V_{ts} = -|V_{ts}| e^{-i\beta_s}. \quad (54)$$

Possible non-standard effects would manifest themselves via $C_{B_q} \neq 1$, $\varphi_{B_d} \neq 0$, or $\varphi_{B_s} \neq 0$. Using this notation the physical observables are

$$\Delta M_q = (\Delta M_q)_{\text{SM}} C_{B_q}, \quad (55)$$

and

$$S_{\psi K_S} = \sin(2\beta + 2\varphi_{B_d}), \quad S_{\psi\phi} = \sin(2|\beta_s| - 2\varphi_{B_s}), \quad (56)$$

with the latter two observables being the coefficients of $\sin(\Delta M_d t)$ and $\sin(\Delta M_s t)$ in the time dependent asymmetries in $B_d^0 \rightarrow \psi K_S$ and $B_s^0 \rightarrow \psi\phi$, respectively. Thus in the presence of non-vanishing φ_{B_d} and φ_{B_s} these two asymmetries do not measure β and β_s but $(\beta + \varphi_{B_d})$ and $(|\beta_s| - \varphi_{B_s})$, respectively.

While working with Wilson coefficients and operator matrix elements at low energy scales is a common procedure, it turns out that for phenomenology it is more useful to work directly with $C_i(\mu_H)$ and with the hadronic matrix elements of the corresponding operators also evaluated at this high scale. The latter matrix elements are given by [41]

$$\langle \bar{K}^0 | Q_i^a | K^0 \rangle = \frac{2}{3} M_K^2 F_K^2 P_i^a(K), \quad (57)$$

where the coefficients $P_i^a(K)$, discussed in more detail below, collect compactly all RG effects from scales below μ_H as well as hadronic matrix elements obtained by lattice methods.

The off-diagonal element in $K^0 - \bar{K}^0$ mixing M_{12}^K is then given by

$$M_{12}^K = \frac{1}{3} M_K F_K^2 \sum_{i,a} C_i^{a*}(\mu_H, K) P_i^a(K). \quad (58)$$

Similarly we find

$$M_{12}^q = \frac{1}{3} M_{B_q} F_{B_q}^2 \sum_{i,a} C_i^{a*}(\mu_H, B_q) P_i^a(B_q). \quad (59)$$

The following points should be emphasized

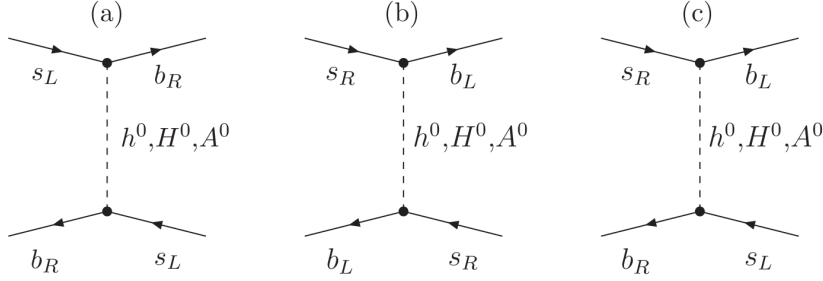


Figure 1: Tree-level Higgs-mediated contributions to $\Delta F = 2$ amplitudes.

- The expressions (58) and (59) are valid for any 2HDM with model dependence entering only the Wilson coefficients $C_i^a(\mu_H)$, which generally also depend on the meson system considered. In particular, they are valid both within and beyond the MFV framework. In MFV models CKM factors and Yukawa couplings define the flavour dependence of these coefficients, while in non-MFV models additional flavour structures are present in $C_i^a(\mu_H)$.
- The coefficients P_i^a are model independent and include the renormalization group evolution from high scale μ_H down to low energy $\mathcal{O}(\mu_K, \mu_B)$. As the physics cannot depend on the renormalization scale μ_H , the P_i^a depend also on μ_H so that the scale dependence present in P_i^a is canceled by the one in C_i^a . Explicit formulae for μ_H dependence of P_i^a can be found in [41]. It should be stressed that here we are talking about logarithmic dependence on μ_H . The power-like dependence (such as $1/M_H^2, \dots$) is present only in the C_i^a .
- The P_i^a depend however on the system considered as the hadronic matrix elements of the operators in (45) relevant for $K^0 - \bar{K}^0$ mixing differ from the matrix elements of analogous operators relevant for $B_{s,d}^0 - \bar{B}_{s,d}^0$ systems. Moreover whereas the RG evolution in the latter systems stops at $\mu_B = \mathcal{O}(M_B)$, in the case of $K^0 - \bar{K}^0$ system it is continued down to $\mu_K \sim 2$ GeV, where the hadronic matrix elements are evaluated by lattice methods.

The advantage of formulating everything at the high energy scale will be evident in the next subsections.

6.2 Neutral-Higgs exchange in $\Delta F = 2$ amplitudes: general case

6.2.1 Basic formulae

Let us next consider the impact of neutral Higgs exchanges on meson-antimeson mixings generated by the general effective Lagrangian in Eq. (17). The neutral Higgs exchanges contribute to M_{12}^K at tree-level through the diagrams in Fig. 1. Evidently (a), (b) and (c) contribute to $C_1^{SLL}(\mu_H)$, $C_1^{SRR}(\mu_H)$ and $C_2^{LR}(\mu_H)$, respectively. In the absence of QCD corrections to diagrams in Fig. 1,

$$C_1^{LR}(\mu_H) = C_2^{SLL}(\mu_H) = C_2^{SRR}(\mu_H) = 0. \quad (60)$$

Still even in this case the presence of Q_1^{LR} , Q_2^{SLL} and Q_2^{SRR} enters in P_1^{LR} , P_2^{SLL} and P_2^{SRR} as discussed below and seen explicitly in the formulae of [41].

In our phenomenological analysis (both within MFV and beyond) we assume the decoupling limit of the heavy Higgs doublet (see Appendix). In this limit the H^0 and A^0 contributions to $C_1^{SLL}(\mu_H)$ and $C_1^{SRR}(\mu_H)$ cancel approximately each other and contribute constructively to $C_2^{LR}(\mu_H)$. This cancellation is not accidental and can be understood in terms of the $SU(2)_L$ structure of the effective operators, since only Q_2^{LR} survives in the $SU(2)_L$ -invariant limit (see e.g. [5, 43]).

By construction, the contribution from h^0 to all the effective operators can be neglected. In this limit, as seen in (18),

$$C_2^{LR}(\mu_H, K) = -\frac{\epsilon_d^2}{c_\beta^2 M_H^2} (\tilde{\Delta}_d)_{21} (\tilde{\Delta}_d)_{12}^* \quad (61)$$

and using (58) we find

$$M_{12}^K = -\frac{1}{3} M_K F_K^2 P_2^{LR}(K) \frac{\epsilon_d^2}{c_\beta^2 M_H^2} (\tilde{\Delta}_d)_{21}^* (\tilde{\Delta}_d)_{12} . \quad (62)$$

Similarly, we find

$$M_{12}^d = -\frac{1}{3} M_{B_d} F_{B_d}^2 P_2^{LR}(B_d) \frac{\epsilon_d^2}{c_\beta^2 M_H^2} (\tilde{\Delta}_d)_{31}^* (\tilde{\Delta}_d)_{13} , \quad (63)$$

$$M_{12}^s = -\frac{1}{3} M_{B_s} F_{B_s}^2 P_2^{LR}(B_s) \frac{\epsilon_d^2}{c_\beta^2 M_H^2} (\tilde{\Delta}_d)_{32}^* (\tilde{\Delta}_d)_{23} . \quad (64)$$

6.2.2 Phenomenology of K^0 - \bar{K}^0 and $B_{s,d}^0$ - $\bar{B}_{s,d}^0$ systems

In the Kaon system the value of $P_2^{LR}(K)$, computed using the formulae in [41], the input values in Table 1 and the hadronic matrix elements in [58], is

$$P_2^{LR}(K) \approx 66 \quad \text{for} \quad \mu_H = 246 \text{ GeV}. \quad (65)$$

This is about two order of magnitude larger than the corresponding factor for the SM operator: $P_1^{VLL}(K) \approx 0.42$. This difference originates from the strong renormalization group enhancement and the chiral enhancement of the scalar operator Q_2^{LR} . Consequently, even a small new physics contribution to $C_2^{LR}(\mu_H)$ can play an important role in the phenomenology.

Using (62) and the input parameters in Table 1 we find then for the contribution of the neutral Higgs exchanges to ε_K :

$$\Delta\varepsilon_K = -7.7 \times 10^{11} \text{ GeV}^2 \frac{\epsilon_d^2}{c_\beta^2 M_H^2} P_2^{LR}(K) \text{Im}[(\tilde{\Delta}_d)_{21}^* (\tilde{\Delta}_d)_{12}] , \quad (66)$$

leading for $P_2^{LR}(K) = 66$ to the bound in (19).

The formulae for M_{12}^q relevant for the $B_{s,d}^0$ - $\bar{B}_{s,d}^0$ systems have been given in (63) and (64). In these cases the RG effects are substantially reduced and the chiral enhancement is basically absent. One finds

$$P_2^{LR}(B_d) \approx P_2^{LR}(B_s) \approx 3.4 \quad \text{for} \quad \mu_H = 246 \text{ GeV}. \quad (67)$$

parameter	value	parameter	value
F_K	$(155.8 \pm 1.7)\text{MeV}$ [44]	$m_s(2\text{ GeV})$	0.105 GeV [45]
F_{B_d}	$(192.8 \pm 9.9)\text{MeV}$ [44]	$m_d(2\text{ GeV})$	0.006 GeV [45]
F_{B_s}	$(238.8 \pm 9.5)\text{MeV}$ [44]	$ V_{ts} $	0.040 ± 0.003 [52]
\hat{B}_K	0.725 ± 0.026 [44]	$ V_{tb} $	1 ± 0.06 [52]
\hat{B}_{B_d}	1.26 ± 0.11 [44]	$ V_{td} _{\text{tree}}$	$(8.3 \pm 0.5) \cdot 10^{-3}$ [52]
\hat{B}_{B_s}	1.33 ± 0.06 [44]	$ V_{us} $	0.2255 ± 0.0019 [45]
M_{B_s}	5.3664 GeV [45]	$ V_{cb} $	$(41.2 \pm 1.1) \times 10^{-3}$ [45]
M_{B_d}	5.2795 GeV [45]	$\sin(2\beta)_{\text{tree}}$	0.734 ± 0.038 [52]
M_K	0.497614 GeV [45]	$\sin(2\beta_s)$	0.038 ± 0.003 [52]
η_{cc}	1.51 ± 0.24 [46]	$\alpha_s(m_Z)$	0.1184 [53]
η_{tt}	0.5765 ± 0.0065 [47]	ΔM_s	$(17.77 \pm 0.12)\text{ ps}^{-1}$ [55]
η_{ct}	0.47 ± 0.04 [48]	ΔM_d	$(0.507 \pm 0.005)\text{ ps}^{-1}$ [55]
η_B	0.551 ± 0.007 [47, 49]	ΔM_K	$(5.292 \pm 0.009) \cdot 10^{-3}\text{ ps}^{-1}$ [45]
ξ	1.243 ± 0.028 [44]	κ_ϵ	0.94 ± 0.02 [42]
$m_c(m_c)$	$(1.268 \pm 0.009)\text{GeV}$ [50]	$\varepsilon_K^{\text{exp}}$	$(2.229 \pm 0.01) \cdot 10^{-3}$ [45]
$m_t(m_t)$	$(163.5 \pm 1.7)\text{GeV}$ [51]	$S_{\psi K_S}^{\text{exp}}$	0.672 ± 0.023 [55]
$m_b(m_b)$	$(4.2 + 0.17 - 0.07)\text{GeV}$ [45]		

Table 1: Values of the input parameters used in our analysis. Additionally, the P_i^a parameters defined in (57) are computed using results from Ref. [56,57]. The subscript “tree” in $|V_{td}|$ and $\sin(2\beta)$ denotes that these inputs are extracted from data using only tree-level observables [52].

This is about a factor of four larger than the corresponding factor for the SM operator: $P_1^{VLL}(B) \approx 0.72$. We then conclude that the bounds on $(\tilde{\Delta}_d)_{31}$ and $(\tilde{\Delta}_d)_{32}$ are substantially weaker than the bound coming from ε_K of Eq. (19).

For completeness, we report here the bounds on the effective FCNC scalar couplings assuming a NP contribution, in magnitude, up to 20% (50%) of the SM B_d (B_s) mixing amplitude:

$$|\epsilon_d| \times \left| (\tilde{\Delta}_d)_{31}^* (\tilde{\Delta}_d)_{13} \right|^{1/2} \lesssim 5 \times 10^{-5} \times \frac{c_\beta M_H}{100\text{ GeV}}, \quad (68)$$

$$|\epsilon_d| \times \left| (\tilde{\Delta}_d)_{32}^* (\tilde{\Delta}_d)_{23} \right|^{1/2} \lesssim 3 \times 10^{-4} \times \frac{c_\beta M_H}{100\text{ GeV}}. \quad (69)$$

Note that if we enforce the RS-GIM structure in (20) and the bound on ϵ_d in (21), derived from ε_K , we obtain theoretical constraints on the effective FCNC couplings which are well below the phenomenological bounds in (68) and (69):

$$|\epsilon_d| \times \left| (\tilde{\Delta}_d)_{31}^* (\tilde{\Delta}_d)_{13} \right|_{\text{RS-GIM}}^{1/2} \lesssim 2 \times 10^{-6} \times \frac{c_\beta M_H}{100\text{ GeV}}, \quad (70)$$

$$|\epsilon_d| \times \left| (\tilde{\Delta}_d)_{32}^* (\tilde{\Delta}_d)_{23} \right|_{\text{RS-GIM}}^{1/2} \lesssim 1 \times 10^{-5} \times \frac{c_\beta M_H}{100\text{ GeV}}. \quad (71)$$

In other words, as anticipated, within the RS-GIM framework the ε_K constraint naturally forbids significant effects in the $B_{s,d}$ systems.

6.3 Neutral-Higgs exchange in $\Delta F = 2$ amplitudes: 2HDM $_{\overline{\text{MFV}}}$

6.3.1 Basic formulae

Using the effective Hamiltonians in Section 4.2 and proceeding like in the general case we find

$$M_{12}^K = -\frac{1}{3}M_K F_K^2 P_2^{LR}(K) \frac{|a_0|^2}{M_H^2} y_s y_d [y_t^2 V_{ts} V_{td}^*]^2, \quad (72)$$

$$M_{12}^d = -\frac{1}{3}M_{B_d} F_{B_d}^2 P_2^{LR}(B_d) \frac{(a_0 + a_1)(a_0^* + a_2^*)}{M_H^2} y_b y_d [y_t^2 V_{tb} V_{td}^*]^2, \quad (73)$$

$$M_{12}^s = -\frac{1}{3}M_{B_s} F_{B_s}^2 P_2^{LR}(B_s) \frac{(a_0 + a_1)(a_0^* + a_2^*)}{M_H^2} y_b y_s [y_t^2 V_{tb} V_{ts}^*]^2. \quad (74)$$

For comparison, we recall that the SM contribution to M_{12}^q reads

$$(M_{12}^q)_{\text{SM}} = \frac{G_F^2}{12\pi^2} M_W^2 (V_{tb} V_{tq}^*)^2 F_{B_q}^2 M_{B_q} \eta_B \hat{B}_{B_q}(B_q) S_0^*(x_t), \quad (75)$$

with the loop function $S_0^*(x_t = m_t^2/M_W^2)^5$ given in [41]. A similar expression holds for the SM top-quark contribution to M_{12}^K , changing accordingly the flavour indices and with the replacement $\eta_B \hat{B}_{B_q} \rightarrow \eta_{tt} \hat{B}_K$ (see Ref. [41] for notations).

6.3.2 Phenomenology of the $K^0-\bar{K}^0$ and $B_{s,d}^0-\bar{B}_{s,d}^0$ systems

Using the basic formulae of above it is straightforward to find the expressions for ε_K , the mass differences ΔM_q and the asymmetries $S_{\psi K_S}$ and $S_{\psi\phi}$.

In the case of ε_K the inclusion of neutral Higgs contributions amounts to the replacement of the SM box function $S_0(x_t)$ by

$$S_K = S_0(x_t) - \frac{64\pi^2}{\hat{B}_K \eta_{tt}} P_2^{LR}(K) |a_0|^2 \frac{m_d m_s}{M_W^2} \frac{m_t^4 t_\beta^2}{M_H^2 v^2}, \quad (76)$$

where we approximated $1/(c_\beta^2 s_\beta^4)$ by t_β^2 .

In order to write down analogous expressions for the box functions S_q relevant for the $B_{s,d}^0-\bar{B}_{s,d}^0$ systems, we introduce

$$T_q = \frac{64\pi^2}{\hat{B}_{B_q} \eta_B} P_2^{LR}(B_q) (a_0^* + a_1^*)(a_0 + a_2) \frac{m_b m_q}{M_W^2} \frac{m_t^4 t_\beta^2}{M_H^2 v^2}, \quad (77)$$

where T_s and T_d are unambiguously connected by the relation

$$T_d = \frac{m_d \hat{B}_{B_s} P_2^{LR}(B_d)}{m_s \hat{B}_{B_d} P_2^{LR}(B_s)} T_s \approx \frac{m_d}{m_s} T_s. \quad (78)$$

Then we simply have

$$S_q = S_0(x_t) - T_q = |S_q| e^{-2i\varphi_{B_q}}, \quad (79)$$

⁵In spite of the fact that the function S_0 is real, we put a $*$ on this function, anticipating a new phase in the effective S function we will obtain in presence of NP effects (see next section).

$S_{\psi K_S}$ and $S_{\psi\phi}$ given in (56) and

$$\Delta M_q = \frac{G_F^2}{6\pi^2} \eta_B M_{B_q} \hat{B}_{B_q} F_{B_q}^2 M_W^2 |V_{tq}|^2 |S_q|. \quad (80)$$

From (78) it follows that

$$\varphi_{B_d} \approx \frac{m_d}{m_s} \varphi_{B_s}. \quad (81)$$

Before presenting the numerical analysis of these formulae, we would like to list the most relevant properties of these results, some of which have already been outlined in Section 4.2:

- The flavour universality of the box function S is broken, with violations governed by the quark masses relevant for the particular system considered: $m_s m_d$, $m_b m_d$ and $m_b m_s$, for the K , B_d and B_s systems, respectively. This opens the possibility of a large impact in B_s mixing solving the problem of the large CP-violation phase hinted by CDF [13] and D0 [14,15]. However, this can happen only with large flavour blind phases, otherwise, with a_i real, the CP asymmetries would not be affected.
- If we try to accommodate a large CP-violating phase in $B_s^0-\bar{B}_s^0$ mixing in this scenario, we find a correlated shift in the relation between $S_{\psi K_S}$ and the CKM phase β . This shift is determined unambiguously by the relation between T_d and T_s in Eq. (78): it contains no free parameters. The shift is such that the prediction of $S_{\psi K_S}$ *decreases* with respect to the SM case at fixed CKM inputs (assuming a large positive value of $S_{\psi\phi}$, as hinted by CDF and D0). This relaxes the existing tension between $S_{\psi K_S}^{\text{exp}}$ and its SM prediction (see Figure 2).
- The new physics contribution to ε_K is tiny and has unique sign, implying a destructive interference with the SM box amplitude. This contribution alone does not improve the agreement between data and prediction for ε_K . However, given the modified relation between $S_{\psi K_S}$ and the CKM phase β , the true value of β extracted in this scenario increases with respect to SM fits. As a result of this modified value of β , also the predicted value for ε_K *increases* with respect to the SM case, resulting in a better agreement with data (see Figure 3).

The numerical analysis of these effects, illustrated in Figure 2 and 3, has been obtained adopting the following two strategies:

Figure 2: We combine the value of $\sin(2\beta)$ determined by tree-level observables (see Table 1) with ε_K (assuming negligible new-physics effects in ε_K) finding a reference value of $\sin(2\beta)$ independent from $S_{\psi K_S}^{\text{exp}}$. The result thus obtained is $\sin(2\beta) = 0.739 \pm 0.036$. We determine the size of T_s , as a function of φ_{B_s} , requiring a deviation of ΔM_s within 10% of its SM value. This fixes unambiguously all our free parameters as function of the CP-violating phase of the B_s mixing amplitude (or the CP-asymmetry $S_{\psi\phi}$). The B_d mixing phase and the CP-asymmetry $S_{\psi K_S}$ are then computed by means of (56) in terms of φ_{B_s} and the reference value of β .

Figure 3: For each value of the B_s mixing amplitude we determine the value of the new-physics phase φ_{B_s} . Also in this case the magnitude of T_s is fixed requiring a deviation of ΔM_s within 10% of its SM value. By means of (56) we then extract $\sin(2\beta)$ from $S_{\psi K_S}^{\text{exp}}$ obtaining a reference value of $\sin(2\beta)$ independent from ε_K . The value of ε_K is then predicted using SM expressions as a function of the new reference value of $\sin(2\beta)$ (which in turn depends on the B_s mixing phase).

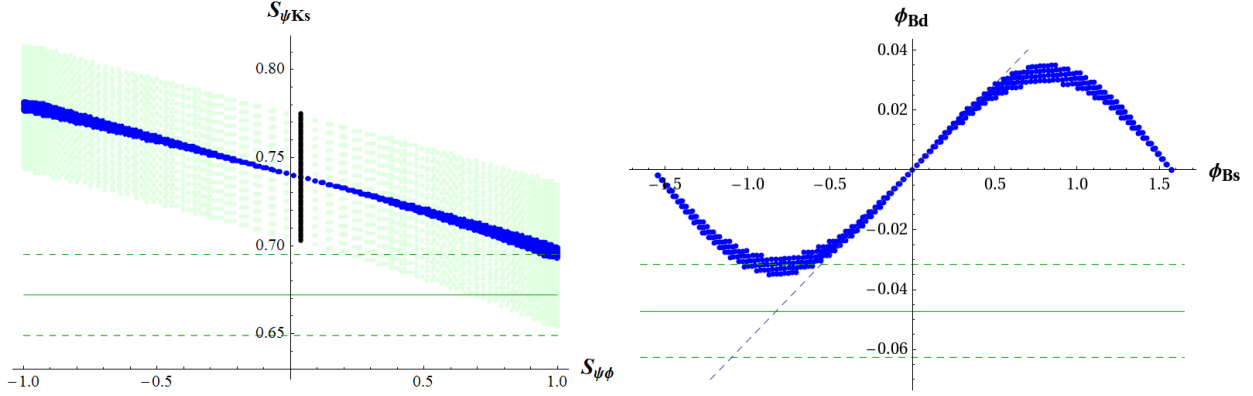


Figure 2: Correlation between $S_{\psi K_S}$ and $S_{\psi\phi}$ (left) and between the new phases in the B_d and B_s mixing (right) originating in the CP-violating Higgs-mediated $\Delta F = 2$ amplitudes in Eqs. (73)–(74). In both plots the blue (dark) points have been obtained with the CKM phase β fixed to its central value ($\sin(2\beta) = 0.739$): the spread is determined only by the condition imposed on ΔM_s (see text). The horizontal lines indicate the $\pm 1\sigma$ range of $S_{\psi K_S}^{\text{exp}}$. On the left plot the $\pm 1\sigma$ error due to the uncertainty in the extraction of β (light points) and the SM prediction (black vertical line) are also shown. The dashed blue (dark) line in the right plot represents $\phi_{B_s} = (m_d/m_s) \phi_{B_s}$.

By looking at the plots in Figure 2 and 3 it is quite clear that a large positive value of $S_{\psi\phi}$ (or a large negative φ_{B_s}), as hinted by CDF [13] and D0 [14, 15], can easily be explained in this framework and, more important, this implies, as a byproduct, a substantial improvement in the predictions of $S_{\psi K_S}$ and ε_K .

In order to understand for which range of the underlying model parameters the desired effect is produced, we list here the conditions of negligible direct impact on ε_K and sizable contribution to B_s mixing. The direct impact of the Higgs-mediated amplitude in ε_K do not exceed the 5% level for

$$|a_0|t_\beta \frac{v}{M_H} < 18, \quad (82)$$

while for

$$\sqrt{|(a_0^* + a_1^*)(a_0 + a_2)|} t_\beta \frac{v}{M_H} = 10 \quad \text{and} \quad \arg [(a_0^* + a_1^*)(a_0 + a_2)] \approx -1.2 \quad (83)$$

we get $S_{\psi\phi} \approx 0.4$, with ΔM_s within 10% of its SM value. As can be noted, the two conditions are perfectly compatible.⁶ The range of free parameters is also very natural, with a_i of order one and t_β moderate ($t_\beta \sim 10$) or large ($t_\beta \sim 50$) depending on the value of M_H . The only condition which is not trivial to achieve is the large CP-violating phase. The latter requires a large difference between a_1 and a_2 that, as already pointed out in Section 4.2, is not easy to obtain in explicit new physics models.

⁶ A large contribution to $B_s^0 - \bar{B}_s^0$ mixing is also compatible with the bounds on t_β and M_H derived by the charged-Higgs exchange in $B \rightarrow \tau\nu$ (see e.g. [60]). These bounds are almost independent from the a_i and can easily be satisfied for $t_\beta = \mathcal{O}(10 \times M_H/v)$. Incidentally, we note that the prediction of $B \rightarrow \tau\nu$ could even improve in this framework because of the higher value of $|V_{ub}|$ extracted from the global analysis of the CKM unitarity triangle, when the Higgs-mediated $\Delta B = 2$ amplitude is taken into account.

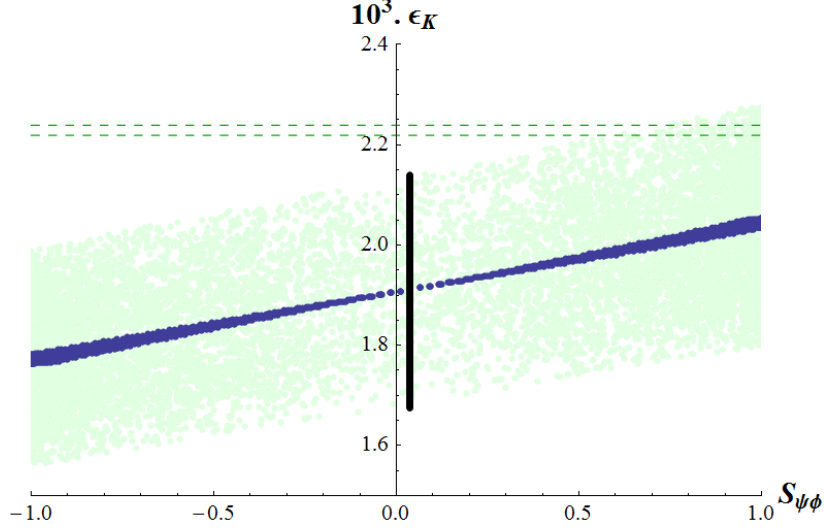


Figure 3: Correlation between ε_K and $S_{\psi K_S}$ with the inclusion of the CP-violating Higgs-mediated $\Delta F = 2$ amplitudes in Eqs. (73)–(74). Notations as in Figure 2.

6.4 The rare decays $B_{s,d} \rightarrow \mu^+ \mu^-$

The rare decays $B_{s,d} \rightarrow \mu^+ \mu^-$ could provide the ultimate and decisive test about magnitude and flavour structure of Higgs-mediated FCNC amplitudes.

Using the effective Hamiltonian in (40) and taking into account the known SM contribution we find

$$\text{Br}(B_q \rightarrow \mu^+ \mu^-) = \text{Br}(B_q \rightarrow \mu^+ \mu^-)_{\text{SM}} \times (|1 + R_q|^2 + |R_q|^2) , \quad (84)$$

where

$$R_q = (a_0^* + a_1^*) \frac{2\pi^2 m_t^2}{Y_0(x_t) M_W^2} \frac{M_{B_q}^2 t_\beta^2}{(1 + m_q/m_b) M_H^2} , \quad (85)$$

and

$$\text{Br}(B_q \rightarrow \mu^+ \mu^-)_{\text{SM}} = \frac{G_F^2 \tau_{B_q}}{\pi} \left(\frac{g^2}{16\pi^2} \right)^2 F_{B_q}^2 M_{B_q} m_\ell^2 \sqrt{1 - \frac{4m_\ell^2}{M_{B_q}^2}} |V_{tb}^* V_{tq}|^2 Y(x_t)^2 , \quad (86)$$

with the loop function $Y(x_t)$ given, for instance, in [54]. In the case of the SM the relation of $\text{Br}(B_q \rightarrow \mu^+ \mu^-)$ to ΔM_q pointed out in [61] allows to reduce the uncertainty and one finds

$$\begin{aligned} \text{Br}(B_d \rightarrow \mu^+ \mu^-)_{\text{SM}} &= (1.0 \pm 0.1) \times 10^{-10} , \\ \text{Br}(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}} &= (3.2 \pm 0.2) \times 10^{-9} . \end{aligned} \quad (87)$$

The striking feature of (85) is the almost exact universality of R_s and R_d : their difference, due to light-quark masses, is well below the parametric uncertainties on the SM predictions for the two branching ratios. This universality, which is not affected by the presence of possible flavour-blind phases, leads to the strict correlation shown in Figure 4. This correlation holds not only for Higgs-mediated amplitudes but, more generally, in the presence of both scalar and

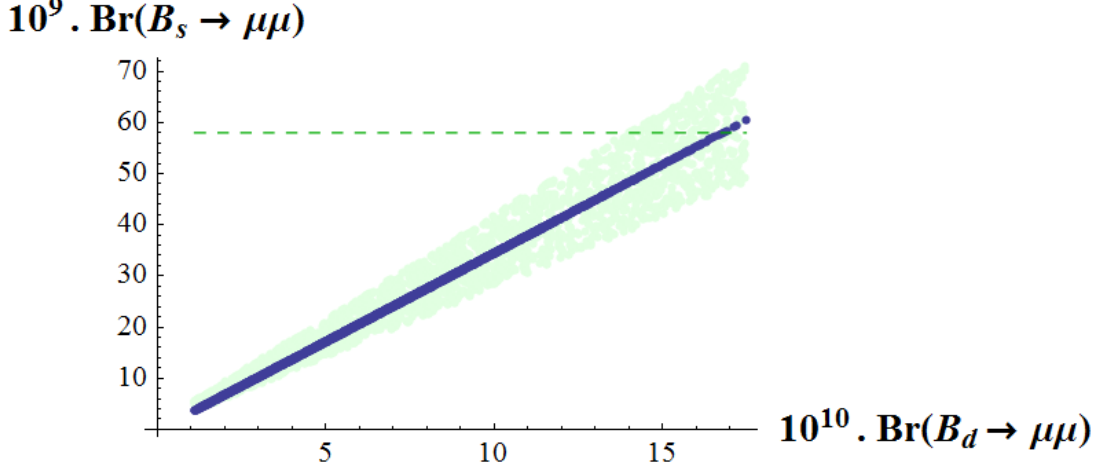


Figure 4: Correlation between $\text{Br}(B_s \rightarrow \mu^+ \mu^-)$ and $\text{Br}(B_d \rightarrow \mu^+ \mu^-)$ in presence of scalar amplitudes respecting the MFV hypothesis. The horizontal dotted line represent the present experimental limit on $\text{Br}(B_s \rightarrow \mu^+ \mu^-)$ from Ref. [59].

$V - A$ operators with a MFV structure [62]. It can indeed be considered as a “smoking-gun” of the MFV hypothesis [62, 63].

Eliminating the dependence of $\text{Br}(B_s \rightarrow \mu^+ \mu^-)/\text{Br}(B_d \rightarrow \mu^+ \mu^-)$ from $|V_{ts}/V_{td}|$ in terms of the corresponding dependence of $\Delta M_s/\Delta M_d$ we can write

$$\frac{\text{Br}(B_s \rightarrow \mu^+ \mu^-)}{\text{Br}(B_d \rightarrow \mu^+ \mu^-)} = \frac{\hat{B}_{B_d} \tau(B_s) \Delta M_s}{\hat{B}_{B_s} \tau(B_d) \Delta M_d} r, \quad r = \frac{M_{B_s}^4 |S_d|}{M_{B_d}^4 |S_s|}, \quad (88)$$

where for $r = 1$ we recover the SM and CMFV relation derived in [61]. In our general MFV framework r can deviate from one; however, this deviation is at most of $\mathcal{O}(10\%)$, as outlined in the previous section. Actually a precise measurement of the two $B_{s,d} \rightarrow \mu^+ \mu^-$ rates would be the best way to determine r and, by means of (88), the amount of non-standard contributions to $\Delta M_s/\Delta M_d$.

This strict correlation of $\text{Br}(B_{s,d} \rightarrow \mu^+ \mu^-)$ shown in Figure 4 should be contrasted with non-MFV frameworks, such as the MSSM with non-minimal flavour structures [22, 23, 54] or models with warped space-time geometry [64]. In some of these frameworks large enhancements of the two $\text{Br}(B_{s,d} \rightarrow \mu^+ \mu^-)$ are possible, but their ratio is no more related to $|V_{ts}/V_{td}|^2$. As a result, the plots corresponding to Figure 4 look very differently (see in particular the plots in [54]).

The upper limit $\text{Br}(B_s \rightarrow \mu^+ \mu^-) < 5.8 \times 10^{-8}$ [59] implies

$$\sqrt{|a_0 + a_1|} t_\beta \frac{v}{M_H} < 8.5. \quad (89)$$

This result is compatible with (83) only if the a_i are of order 1 (for $a_i \ll 1$ it would require an unnaturally large value for a_2/a_1). For $a_i = \mathcal{O}(1)$ it signals that $\text{Br}(B_s \rightarrow \mu^+ \mu^-)$ has to be close to its experimental bound if scalar amplitudes play a significant role in B_s mixing.

7 Conclusions

In the present paper we have addressed the question of the effectiveness of the NFC and MFV hypotheses in suppressing the strength of FCNC transitions to the observed level in models with more than one-Higgs doublet. More generally, we have analysed the interplay between continuous and discrete flavour-blind symmetries and the symmetry-breaking pattern in the flavour sector, in determining the structure of scalar FCNCs. We have considered explicitly only a general 2HDM, but our discussion applies also to models with more than two-Higgs doublets.

The NFC hypothesis is based on the imposition of flavour-blind symmetries in the Yukawa Lagrangian which do not hold beyond the tree-level. On the contrary, the MFV hypotheses is based on a symmetry and symmetry-breaking pattern in the flavour sector which is renormalization group invariant. As a result, it is not surprising that the latter framework turns out to be superior. As we have shown, this is evident when effects beyond the tree-level are taken into account. As we have illustrated with the example of ε_K , beyond the tree-level the NFC hypothesis ceases to provide a sufficient suppression of FCNCs in a natural manner, unless the neutral Higgs masses are well above the LHC energy scales. This should be contrasted with MFV framework, which is stable under higher order contributions. Within the latter framework relatively low values for the Higgs masses can be accommodated, as well as large values of the bottom Yukawa coupling (namely $t_\beta \gg 1$). The comparison between NFC and MFV has also given us the opportunity to clarify which of the multi-Higgs models proposed in the recent literature are consistent with the MFV principle, and thus are naturally protected against too large FCNCs.

The MFV hypothesis is very simple, theoretically sound and, as we have just stressed, very efficient in suppressing large FCNC contributions to the measured level. Yet, the important phenomenological question remains whether such a constrained framework is at the end consistent with the nature around us. Indeed, there exist at least three anomalies observed in the data that, at first sight, give a clear hint for the presence of non-MFV interactions. These are as follows.

- First of all the size of the CP-violation in $B_s^0-\bar{B}_s^0$ system signaled by the CP-asymmetry $S_{\psi\phi}$ in $B_s \rightarrow \psi\phi$ observed by CDF and D0 that appears to be roughly by a factor of 20 larger than the SM and MFV predictions, assuming the Yukawa couplings to be the only sources of CP-violation.
- The value of $\sin 2\beta$ resulting from the UT fits tends to be significantly larger than the measured value of $S_{\psi K_S}$.
- The value of ε_K predicted in the SM by using $S_{\psi K_S}$ as the measure of the observed CP-violation is about 2σ lower than the data. In short, the values of $S_{\psi K_S}$ and ε_K cannot be simultaneously described within the SM [19, 20].

As pointed out in Ref. [16–18], the mechanisms of flavour and CP-violation do not necessarily need to be related. In particular, as noted in [16], a large new phase in $B_s^0-\bar{B}_s^0$ mixing could in principle be obtained in the MFV framework if additional flavour-blind phases are present. This idea cannot be realized in the ordinary MSSM with MFV, as shown in [54].⁷ However, it could

⁷ As discussed in Section 4.2, the difficulty of realizing this scenario in the MSSM is due to the suppression in the MSSM of effective operators with several Yukawa insertions. Sizable couplings for these operators are necessary both to have an effective large CP-violating phase in $B_s^0-\bar{B}_s^0$ mixing and, at the same time, to evade bounds from other

be realized in different underlying models, such as the up-lifted MSSM, as recently pointed out in Ref. [24].

In the present work we have demonstrated that a general 2HDM with MFV, enriched by flavour blind phases (2HDM_{MFV}), is not only capable in explaining the first anomaly (large $S_{\psi\phi}$): once the first problem is addressed, unique solutions to the other two problems listed above ($S_{\psi K_s} - \varepsilon_K$) are naturally at work. Indeed, a small new phase in $B_d^0 - \bar{B}_d^0$ system with the correct sign and roughly correct size is automatically implied by a large phase in the $B_s^0 - \bar{B}_s^0$ system with the hierarchy of these two new phases being fixed by the ratio m_d/m_s . In practice this solves (or at least softens) the remaining two problems by enhancing the true value of the phase β of V_{td} : the values of $S_{\psi K_s}$ and ε_K thus obtained are in better agreement with expectations in spite of the negligible contribution of neutral Higgs exchanges to ε_K . In summary, the neutral Higgs exchanges contributing to $\Delta F = 2$ processes in the 2HDM_{MFV} seems to be an interesting solution to all these anomalies, with a clear pattern of correlations that could be easily verified or falsified in the near future.

It must be stressed that this mechanisms and pattern of correlations are quite different than other mechanisms proposed in the recent literature to accommodate a large $B_s^0 - \bar{B}_s^0$ mixing phase. Those models typically contain more free parameters (associated to new flavour-breaking sources) and do not provide a natural explanation of why the deviations from the SM should be small (or vanishingly small) in the $B_d^0 - \bar{B}_d^0$ mixing and in ε_K .

Finally, we have stressed the key role of the rare decays $B_{s,d} \rightarrow \mu^+ \mu^-$ in providing a decisive test of the flavour-breaking structure implied by MFV, independently of possible flavour-blind phases, and independently of the dominance of scalar vs. vector FCNC operators beyond the SM.

Acknowledgments

We thank Wolfgang Altmannshofer, Monika Blanke, and Toni Pich for useful comments and discussions. AJB would like to thank Gustavo Branco for bringing to his attention Ref. [10], which motivated us the reconsideration of MFV in multi-Higgs models. AJB, SG, and GI thank the Galileo Galilei Institute for Theoretical Physics for the hospitality and partial support during the completion of this work. This research was partially supported by the Cluster of Excellence ‘Origin and Structure of the Universe’, by the Graduiertenkolleg GRK 1054 of DFG, by the German ‘Bundesministerium für Bildung und Forschung’ under contract 05H09WOE, and by the EU Marie Curie Research Training Network contracts MTRN-CT-2006-035482 (*Flavianet*) and MRTN-CT-2006-035505 (*HEP-TOOLS*).

Added note

After the completion of this work, two papers discussing related issues have appeared: a model-independent analysis of new-physics effects in B_s and B_d mixing [66], and a detailed analysis of flavour-changing amplitudes mediated by charged-Higgs exchange [67].

observables, such as $B_s \rightarrow \mu^+ \mu^-$ and $B \rightarrow X_s \gamma$.

Appendix

The Higgs Lagrangian of a generic model with two-Higgs doublets, H_1 and H_2 , with hypercharges $Y = 1/2$ and $Y = -1/2$ respectively, can be decomposed as

$$\mathcal{L}_{\text{Higgs}}^{2\text{HDM}} = \sum_{i=1,2} D_\mu H_i D_\mu H_i^\dagger + \mathcal{L}_Y - V(H_1, H_2) , \quad (90)$$

where $D_\mu H_i = \partial_\mu H_i - ig'Y \hat{B}_\mu H_i - igT_a \hat{W}_\mu^a H_i$, with $T_a = \tau_a/2$. The potential is such that the H_i gets a non-trivial vev, giving rise to non-vanishing masses for M_W and M_Z bosons. In the unitary gauge we can set

$$\langle H_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix} , \quad \langle H_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_2 \\ 0 \end{pmatrix} , \quad v^2 = v_1^2 + v_2^2 \approx 246 \text{ (GeV)}^2 , \quad (91)$$

where v_1 and v_2 can be always taken positive, and we assume their phases to be zero in order to avoid spontaneous CP breaking.

A useful change of basis is obtained with the following global rotation

$$\begin{pmatrix} \Phi_v \\ \Phi_H \end{pmatrix} = \frac{1}{v} \begin{pmatrix} v_1 & v_2 \\ -v_2 & v_1 \end{pmatrix} \begin{pmatrix} H_1 \\ H_2^c \end{pmatrix} . \quad (92)$$

In the new basis only the doublet Φ_v has a non-vanishing vev, and the eight degrees of freedom of the two-Higgs doublets appear explicitly as three Goldstone bosons G^\pm and G^0 , two charged Higgs G^\pm , and three neutral scalars $S_{1,2,3}$:

$$\Phi_v = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + S_1 + iG^0) \end{pmatrix} , \quad \Phi_H = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(S_2 + iS_3) \end{pmatrix} . \quad (93)$$

It is worth to stress that, in the absence of an interaction distinguishing the two-Higgs fields, the parameter $t_\beta \equiv \tan(\beta) = v_2/v_1$ is not well defined: we can always mix the two fields by means of rotations of the type (92). This is not the case if we assume an exact $U(1)_{\text{PQ}}$ symmetry (or its discrete Z_2 subgroup), which would allows us to distinguish the two fields. Throughout this paper we assume the $U(1)_{\text{PQ}}$ breaking terms in the Higgs potential are calculable (or identifiable) from first principles (e.g. the $U(1)_{\text{PQ}}$ symmetry is only softly broken, as in the MSSM), such that the two fields can be defined starting from the $U(1)_{\text{PQ}}$ limit of the theory (a detailed discussion about the definition of t_β beyond the tree-level in the MSSM can be found in Ref. [43]).

The most general potential for the two-Higgs doublets that is renormalizable and gauge invariant is

$$\begin{aligned} V(H_1, H_2) = & \mu_1^2 |H_1|^2 + \mu_2^2 |H_2|^2 + (bH_1 H_2 + \text{h.c.}) + \frac{\lambda_1}{2} |H_1|^4 + \frac{\lambda_2}{2} |H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 \\ & + \lambda_4 |H_1 H_2|^2 + \left[\frac{\lambda_5}{2} (H_1 H_2)^2 + \lambda_6 |H_1|^2 H_1 H_2 + \lambda_7 |H_2|^2 H_1 H_2 + \text{h.c.} \right] , \end{aligned} \quad (94)$$

where $H_1 H_2 = H_1^T (i\sigma_2) H_2$ and all the parameters must be real with the exception of b and $\lambda_{5,6,7}$. Changing the relative phase of H_1 and H_2 we can cancel the phase of b and $\lambda_{6,7}$ relative to λ_5 . As a result, the potential is invariant under CP if b and $\lambda_{5,6,7}$ are all real, if only one of these couplings is different from zero, or if their phases are related [$\arg(\lambda_5) = 2 \arg(b) = 2 \arg(\lambda_{6,7})$].

In several explicit models the coefficients $\lambda_{6,7}$ are set to zero. This can be achieved by imposing a discrete Z_2 symmetry that is only softly broken by the terms proportional to b and λ_5 . Both b and λ_5 break the $U(1)_{PQ}$ symmetry and, if $\lambda_6 = \lambda_7 = 0$, at least one of them must be non-zero to prevent the appearance of a massless pseudoscalar Goldstone boson.

In order to analyse the spectrum of the theory, let us first restrict the attention to the case of exact CP invariance. In this case the neutral mass eigenstates are two CP-even (h^0 and H^0) and one CP-odd (A^0) states. The masses of the CP-odd and charged fields are

$$M_A^2 = \frac{b}{s_\beta c_\beta} - \frac{1}{2}v^2 \left(2\lambda_5 - \lambda_6 t_\beta^{-1} - \lambda_7 t_\beta \right) , \quad (95)$$

$$M_{H^\pm}^2 = M_A^2 + \frac{1}{2}v^2 (\lambda_5 - \lambda_4) . \quad (96)$$

The two CP-even states mix according with the squared-mass matrix

$$\mathcal{M}^2 = M_A^2 \begin{pmatrix} s_\beta^2 & -s_\beta c_\beta \\ -s_\beta c_\beta & c_\beta^2 \end{pmatrix} + \mathcal{B}^2 , \quad (97)$$

$$\mathcal{B}^2 = \begin{pmatrix} \lambda_1 c_\beta^2 + \lambda_5 s_\beta^2 - 2\lambda_6 s_\beta c_\beta & (\lambda_3 + \lambda_4) s_\beta c_\beta - \lambda_6 c_\beta^2 - \lambda_7 s_\beta^2 \\ (\lambda_3 + \lambda_4) s_\beta c_\beta - \lambda_6 c_\beta^2 - \lambda_7 s_\beta^2 & \lambda_2 s_\beta^2 + \lambda_5 c_\beta^2 - 2\lambda_7 s_\beta c_\beta \end{pmatrix} . \quad (98)$$

whose eigenstates are

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos(\alpha - \beta) & \sin(\alpha - \beta) \\ -\sin(\alpha - \beta) & \cos(\alpha - \beta) \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \end{pmatrix} , \quad (99)$$

$$m_{H^0, h^0}^2 = \frac{1}{2} \left[\mathcal{M}_{11}^2 + \mathcal{M}_{22}^2 \pm \sqrt{(\mathcal{M}_{11}^2 - \mathcal{M}_{22}^2)^2 + 4(\mathcal{M}_{12}^2)^2} \right] , \quad (100)$$

The CP-even eigenstates are defined such that $M_{h^0} < M_{H^0}$, and the explicit expression of the mixing angle α is

$$\tan(2\alpha) = \frac{2\mathcal{M}_{12}^2}{\mathcal{M}_{11}^2 - \mathcal{M}_{22}^2} . \quad (101)$$

As discussed in [65], for $M_A^2 \gg |\lambda_i|v^2$ we are in the decoupling regime where

$$M_{h^0} = \mathcal{O}(v) , \quad M_{H^0}, M_{H^\pm} = M_A + \mathcal{O}\left(\frac{v^2}{M_A}\right) , \quad \cos(\beta - \alpha) = \mathcal{O}\left(\frac{v^2}{M_A^2}\right) . \quad (102)$$

This regime is characterised by a negligible mass mixing of the two doublets Φ_v and Φ_H , and by two separate mass scales ($M_A \gg M_{h^0}$).

As can be seen from (96), the decoupling limit is naturally realized if $b = \mathcal{O}(\lambda_i v^2)$ and $t_\beta \gg 1$ or even if $b \gg \lambda_i v^2$ and $t_\beta = \mathcal{O}(1)$. The decoupling regime cannot be realized if $b = \lambda_6 = \lambda_7 = 0$. However, this limit is not particularly interesting for our purposes since in this case we cannot reach large values of t_β and, at the same time, be compatible with the LEP bounds on M_{h^0} .

Finally, let us briefly discuss the possibility of CP-violation, in the simplified limit where $\lambda_3 = \lambda_4 = \lambda_6 = \lambda_7 = 0$. In this case only b and λ_5 can be complex, but we can always rotate the Higgs fields such that b is real. The spectrum contains a charged Higgs, with mass

$$M_{H^\pm}^2 = \frac{b}{c_\beta s_\beta} - \frac{\text{Re}(\lambda_5)}{2}v^2 , \quad (103)$$

and three scalar particles linear combinations of $S_{1,2,3}$. In particular, considering the large t_β regime, we can expand for small c_β and obtain at the zeroth order

$$M_1^2 \sim \lambda_2 v^2, \quad (104)$$

$$M_2^2 \sim \frac{b}{c_\beta} - \frac{v^2}{2} (\text{Re}(\lambda_5) - |\lambda_5|), \quad (105)$$

$$M_3^2 \sim \frac{b}{c_\beta} - \frac{v^2}{2} (\text{Re}(\lambda_5) + |\lambda_5|), \quad (106)$$

from where we can notice the approximate degeneracy of the charged Higgs and the two scalars of mass M_2 and M_3 . Denoting with h_1, h_2, h_3 the corresponding mass eigenstates, they can be expressed in terms of the original fields S_1, S_2, S_3 as

$$h_1 \propto \left(-\frac{2b}{3v^2 c_\beta^2}, \frac{2}{3}(\lambda_2 - \text{Re}(\lambda_5)), \text{Im}(\lambda_5) \right), \quad (107)$$

$$h_2 \propto \left(\frac{v^2 c_\beta^2}{2b} (\text{Re}(\lambda_5) - 3|\lambda_5| + 2\lambda_2), 1, \frac{\text{Re}(\lambda_5) - |\lambda_5|}{\text{Im}(\lambda_5)} \right), \quad (108)$$

$$h_3 \propto \left(\frac{v^2 c_\beta^2}{2b} F(\lambda_2, \lambda_5), (\text{Re}(\lambda_5) + |\lambda_5|)^2, \frac{(\text{Re}(\lambda_5) + |\lambda_5|)^3}{\text{Im}(\lambda_5)} \right), \quad (109)$$

where the $F(\lambda_2, \lambda_5)$ function has a finite limit for $\text{Im}(\lambda_5) \rightarrow 0$. Two comments are in order: (1) for $t_\beta \gg 1$ we obtain the decoupling of S_1 from $S_{2,3}$, namely we are in the decoupling regime where the mass mixing of the two doublets Φ_v and Φ_H is negligible; (2) the mixing of S_2 and S_3 is large even if $t_\beta \gg 1$, and vanishes only in the limit where $\text{Im}(\lambda_5) \ll \text{Re}(\lambda_5)$, namely in the limit of approximate CP conservation.

References

- [1] S. L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D **2** (1970) 1285.
- [2] S. L. Glashow and S. Weinberg, Phys. Rev. D **15** (1977) 1958.
- [3] E. A. Paschos, Phys. Rev. D **15** (1977) 1966.
- [4] A. J. Buras, P. Gambino, M. Gorbahn, S. Jager and L. Silvestrini, Phys. Lett. B **500** (2001) 161 [arXiv:hep-ph/0007085].
- [5] G. D'Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, Nucl. Phys. B **645** (2002) 155 [arXiv:hep-ph/0207036].
- [6] R. S. Chivukula and H. Georgi, Phys. Lett. B **188** (1987) 99.
- [7] L. J. Hall and L. Randall, Phys. Rev. Lett. **65** (1990) 2939.
- [8] A. S. Joshipura and B. P. Kodrani, Phys. Rev. D **77** (2008) 096003 [arXiv:0710.3020 [hep-ph]]; Phys. Rev. D **81** (2010) 035013 [arXiv:0909.0863 [hep-ph]].
- [9] A. Pich and P. Tuzon, Phys. Rev. D **80** (2009) 091702 [arXiv:0908.1554 [hep-ph]]; arXiv:1001.0293.
- [10] F. J. Botella, G. C. Branco and M. N. Rebelo, Phys. Lett. B **687** (2010) 194 [arXiv:0911.1753].
- [11] R. S. Gupta and J. D. Wells, Phys. Rev. D **81** (2010) 055012 [arXiv:0912.0267 [hep-ph]].
- [12] P. M. Ferreira, L. Lavoura and J. P. Silva, arXiv:1001.2561.

- [13] T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **100** (2008) 161802 [arXiv:0712.2397 [hep-ex]].
- [14] V. M. Abazov *et al.* [D0 Collaboration], arXiv:1005.2757.
- [15] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **101** (2008) 241801 [arXiv:0802.2255 [hep-ex]].
- [16] A. L. Kagan, G. Perez, T. Volansky and J. Zupan, Phys. Rev. D **80** (2009) 076002 [arXiv:0903.1794 [hep-ph]].
- [17] L. Mercolli and C. Smith, Nucl. Phys. B **817** (2009) 1 [arXiv:0902.1949 [hep-ph]].
- [18] P. Paradisi and D. M. Straub, Phys. Lett. B **684** (2010) 147 [arXiv:0906.4551 [hep-ph]].
- [19] E. Lunghi and A. Soni, Phys. Lett. B **666** (2008) 162 [arXiv:0803.4340 [hep-ph]].
- [20] A. J. Buras and D. Guadagnoli, Phys. Rev. D **78**, 033005 (2008) [arXiv:0805.3887 [hep-ph]].
- [21] R. D. Peccei and H. R. Quinn, Phys. Rev. D **16** (1977) 1791.
- [22] G. Isidori and A. Retico, JHEP **0209** (2002) 063 [arXiv:hep-ph/0208159].
- [23] P. H. Chankowski and L. Slawianowska, Phys. Rev. D **63**, 054012 (2001) [arXiv:hep-ph/0008046].
- [24] B. A. Dobrescu, P. J. Fox and A. Martin, arXiv:1005.4238 [hep-ph].
- [25] K. Agashe, G. Perez and A. Soni, Phys. Rev. D **71**, 016002 (2005) [arXiv:hep-ph/0408134]; C. Csaki, A. Falkowski and A. Weiler, JHEP **0809** (2008) 008 [arXiv:0804.1954 [hep-ph]].
- [26] R. Contino, T. Kramer, M. Son and R. Sundrum, JHEP **0705** (2007) 074 [arXiv:hep-ph/0612180].
- [27] S. Davidson, G. Isidori and S. Uhlig, Phys. Lett. B **663**, 73 (2008) [arXiv:0711.3376 [hep-ph]].
- [28] M. Blanke, A. J. Buras, B. Duling, S. Gori and A. Weiler, JHEP **0903** (2009) 001 [arXiv:0809.1073 [hep-ph]].
- [29] G. F. Giudice and O. Lebedev, Phys. Lett. B **665** (2008) 79 [arXiv:0804.1753 [hep-ph]].
- [30] K. Agashe and R. Contino, Phys. Rev. D **80** (2009) 075016 [arXiv:0906.1542 [hep-ph]].
- [31] A. Azatov, M. Toharia and L. Zhu, Phys. Rev. D **80** (2009) 035016 [arXiv:0906.1990 [hep-ph]].
- [32] P. Paradisi, M. Ratz, R. Schieren and C. Simonetto, Phys. Lett. B **668** (2008) 202 [arXiv:0805.3989 [hep-ph]].
- [33] G. Colangelo, E. Nikolidakis and C. Smith, Eur. Phys. J. C **59** (2009) 75 [arXiv:0807.0801 [hep-ph]].
- [34] T. Feldmann and T. Mannel, Phys. Rev. Lett. **100** (2008) 171601 [arXiv:0801.1802 [hep-ph]].
- [35] K. S. Babu and C. F. Kolda, Phys. Rev. Lett. **84** (2000) 228 [arXiv:hep-ph/9909476].
- [36] G. Isidori and A. Retico, JHEP **0111** (2001) 001 [arXiv:hep-ph/0110121].
- [37] A. J. Buras, P. H. Chankowski, J. Rosiek and L. Slawianowska, Nucl. Phys. B **659** (2003) 3 [arXiv:hep-ph/0210145].
- [38] A. Dedes and A. Pilaftsis, Phys. Rev. D **67** (2003) 015012 [arXiv:hep-ph/0209306].
- [39] G. C. Branco, W. Grimus and L. Lavoura, Phys. Lett. B **380** (1996) 119 [arXiv:hep-ph/9601383].
- [40] A. J. Buras, G. Isidori and P. Paradisi, arXiv:1007.5291 [hep-ph].
- [41] A. J. Buras, S. Jager and J. Urban, Nucl. Phys. B **605** (2001) 600 [arXiv:hep-ph/0102316].
- [42] A. J. Buras, D. Guadagnoli and G. Isidori, Phys. Lett. B **688** (2010) 309 [arXiv:1002.3612 [hep-ph]].
- [43] M. Gorbahn, S. Jager, U. Nierste and S. Trine, arXiv:0901.2065 [hep-ph].
- [44] J. Laiho, E. Lunghi and R. S. Van de Water, Phys. Rev. D **81** (2010) 034503 [arXiv:0910.2928 [hep-ph]].

- [45] C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667** (2008) 1.
- [46] S. Herrlich and U. Nierste, Nucl. Phys. B **419** (1994) 292 [arXiv:hep-ph/9310311].
- [47] A. J. Buras, M. Jamin and P. H. Weisz, Nucl. Phys. B **347** (1990) 491.
- [48] S. Herrlich and U. Nierste, Phys. Rev. D **52** (1995) 6505 [arXiv:hep-ph/9507262].
- [49] G. Buchalla, Phys. Lett. B **395** (1997) 364 [arXiv:hep-ph/9608232].
- [50] I. Allison *et al.* [HPQCD Collaboration], Phys. Rev. D **78** (2008) 054513 [arXiv:0805.2999 [hep-lat]].
- [51] [Tevatron Electroweak Working Group and CDF Collaboration and D0 Collab], arXiv:0903.2503 [hep-ex].
- [52] M. Bona *et al.* [UTfit Collaboration], JHEP **0803** (2008) 049 [arXiv:0707.0636 [hep-ph]]. Updates available on <http://www.utfit.org>.
- [53] S. Bethke, Eur. Phys. J. C **64** (2009) 689 [arXiv:0908.1135 [hep-ph]].
- [54] W. Altmannshofer, A. J. Buras, S. Gori, P. Paradisi and D. M. Straub, Nucl. Phys. B **830** (2010) 17 [arXiv:0909.1333 [hep-ph]].
- [55] E. Barberio *et al.* [Heavy Flavor Averaging Group], arXiv:0808.1297 [hep-ex].
- [56] C. R. Allton *et al.*, Phys. Lett. B **453** (1999) 30 [arXiv:hep-lat/9806016].
- [57] D. Becirevic, V. Gimenez, G. Martinelli, M. Papinutto and J. Reyes, JHEP **0204** (2002) 025 [arXiv:hep-lat/0110091].
- [58] R. Babich, N. Garron, C. Hoelbling, J. Howard, L. Lellouch and C. Rebbi, Phys. Rev. D **74**, 073009 (2006) [arXiv:hep-lat/0605016].
- [59] T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **100** (2008) 101802 [arXiv:0712.1708 [hep-ex]].
- [60] M. Bona *et al.* [UTfit Collaboration], Phys. Lett. B **687** (2010) 61 [arXiv:0908.3470 [hep-ph]].
- [61] A. J. Buras, Phys. Lett. B **566** (2003) 115 [arXiv:hep-ph/0303060].
- [62] T. Hurth, G. Isidori, J. F. Kamenik and F. Mescia, Nucl. Phys. B **808** (2009) 326 [arXiv:0807.5039 [hep-ph]].
- [63] M. Blanke, A. J. Buras, D. Guadagnoli and C. Tarantino, JHEP **0610** (2006) 003 [arXiv:hep-ph/0604057].
- [64] M. Blanke, A. J. Buras, B. Duling, K. Gemmler and S. Gori, JHEP **0903** (2009) 108 [arXiv:0812.3803 [hep-ph]].
- [65] J. F. Gunion and H. E. Haber, Phys. Rev. D **67** (2003) 075019 [arXiv:hep-ph/0207010].
- [66] Z. Ligeti, M. Papucci, G. Perez and J. Zupan, arXiv:1006.0432.
- [67] M. Jung, A. Pich and P. Tuzon, arXiv:1006.0470.